

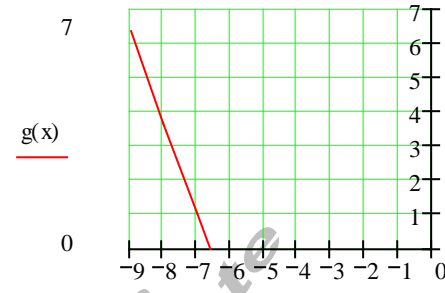
Aufgaben: Lineare Ungleichungen und Lineare Optimierung.

1. Stellen Sie die folgenden Ungleichungen nach y um:

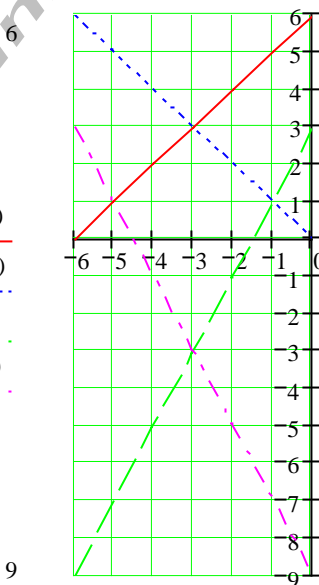
a) $\frac{1}{2}x \leq 5 - 3y + 12$	b) $\frac{1}{4}y + \frac{3}{4}x - 12 \geq \frac{3}{4}y - \frac{1}{4}x - 15$	c) $-y + \frac{2}{3}x + \frac{2}{5} \geq 3 + \frac{1}{2}y - \frac{1}{3}x$
Lösung		
a) $y \leq -\frac{1}{6}x + \frac{17}{3}$	b) $y \leq 2x + 6$	c) $y \leq \frac{2}{3}x - \frac{26}{15}$

2. Bestimmen Sie den Graphen der Lösungsmenge der linearen Ungleichungen:

a) $3x + 2y + 6 < 0$	$D = \{ x \mid -3 \leq x \leq 2 \}$	$W = \{ y \mid -6 \leq y \leq 2 \}$
Lösung		
$y < -\frac{3}{2}x - 3$	$g(x)$	
b) $\frac{1}{2}x \geq \frac{1}{2}y - 3$	$D = \{ x \mid -6 \leq x \leq 0 \}$	$W = \{ y \mid 0 \leq y \leq 6 \}$
Lösung		
$y \leq x + 6$	$g(x)$	

$c) \frac{8}{3}x \geq -y - 17 \frac{2}{3}$	$D = \{ x \mid -9 \leq x \leq 0 \}$	$W = \{ y \mid 0 \leq y \leq 7 \}$
Lösung		
$y \geq -\frac{8}{3}x - 17 \frac{2}{3}$		

3. Bestimmen Sie den Graphen der Lösungsmenge des linearen Ungleichungssystems und seine Eckpunkte.

$3 a) \begin{cases} \frac{1}{2}x \geq \frac{1}{2}y - 3 \wedge \\ -3x \geq 3y \wedge \\ x \leq \frac{1}{2}y - \frac{3}{2} \wedge \\ 2y \geq -4x - 18 \end{cases}$	$D = \{ x \mid -6 \leq x \leq 0 \}$ $W = \{ y \mid -9 \leq y \leq 6 \}$
Lösung	
<p>U1: $y \leq x + 6$ U2: $y \leq -x$ U3: $y \geq 2x + 3$ U4: $y \geq -2x - 9$</p> <p>A(-5 1) B(-3 -3) C(-1 1) D(-3 3)</p>	

$-\frac{1}{2}x \geq -3 + \frac{1}{2}y \wedge$ <p>3 b) $\frac{1}{2}x \geq \frac{1}{2}y \wedge$</p> $2y \geq 6 - 4x \wedge$ $x \leq \frac{1}{2}y + 4 \frac{1}{2}$	$D = \{ x \mid 0 \leq x \leq 6 \}$ $W = \{ y \mid -9 \leq y \leq 6 \}$
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Lösung

<p>U1: $y \leq -x + 6$</p> <p>U2: $y \leq x$</p> <p>U3: $y \geq -2x + 3$</p> <p>U4: $y \geq 2x - 9$</p> <p>A (1 1)</p> <p>B (3 -3)</p> <p>C (5 1)</p> <p>D (3 3)</p>	
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<p>3 c) $-x \geq 6 - 3y \wedge$</p> $\frac{1}{5}x \geq \frac{1}{5}y - 1 \wedge$ $x \leq -\frac{1}{3}y - 1$	$D = \{ x \mid -6 \leq x \leq 0 \}$ $W = \{ y \mid -3 \leq y \leq 5 \}$
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Lösung

<p>U1: $y \geq \frac{1}{3}x + 2$</p> <p>U2: $y \leq x + 5$</p> <p>U3: $y \leq -3x - 3$</p> <p>A $\left(-4 \frac{1}{2} \mid \frac{1}{2} \right)$</p> <p>B $\left(-1 \frac{1}{2} \mid 1 \frac{1}{2} \right)$</p> <p>C (-2 3)</p>	
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$x \geq -3y + 6 \wedge$ $3 \text{ d) } \frac{1}{5}x - 1 \leq -\frac{1}{5}y \wedge$ $1 - x \leq -\frac{1}{3}y$	$D = \{ x \mid 0 \leq x \leq 6 \}$ $W = \{ y \mid -3 \leq y \leq 5 \}$
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Lösung

$U1: y \geq -\frac{1}{3}x + 2$ $U2: y \leq -x + 5$ $U3: y \leq 3x - 3$ $A \left(1 \frac{1}{2} \mid 1 \frac{1}{2} \right)$ $B \left(4 \frac{1}{2} \mid \frac{1}{2} \right)$ $C(2 \mid 3)$	
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$y \geq x - 2 \wedge$ $3 \text{ e) } 2x + y \geq 10 \wedge$ $2x + 3y \leq 24$	$D = \{ x \mid 0 \leq x \leq 6 \}$ $W = \{ y \mid 0 \leq y \leq 7 \}$
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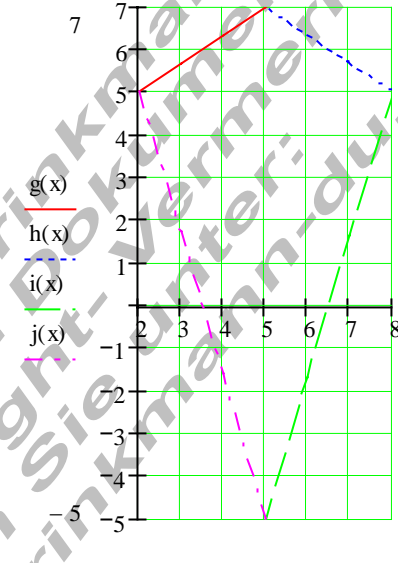
Lösung

$U1: y \geq x - 2$ $U2: y \geq -2x + 10$ $U3: y \leq -\frac{2}{3}x + 8$ $A \left(1 \frac{1}{2} \mid 7 \right)$ $B(4 \mid 2)$ $C(6 \mid 4)$	
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$y + \frac{2}{3}x \geq \frac{11}{3} \wedge$ $3f) y - \frac{2}{5}x \geq 1 \wedge$ $\frac{2}{7}x + y \leq 4$	$D = \{ x \mid -2 \leq x \leq 5 \}$ $W = \{ y \mid 0 \leq y \leq 5 \}$
Lösung	
$U1: y \geq -\frac{2}{3}x + \frac{11}{3}$ $U2: y \geq \frac{2}{5}x + 1$ $U3: y \leq -\frac{2}{7}x + 4\frac{3}{7}$ $A(-2 \mid 5)$ $B\left(2\frac{1}{2} \mid 2\right)$ $C(5 \mid 3)$	

$-y \geq -\frac{5}{7}x - 11\frac{1}{14} \wedge$ $-y \geq \frac{5}{7}x - 3\frac{13}{14} \wedge$ <p>3 g) $\frac{8}{3}x \geq -y - 17\frac{2}{3} \wedge$</p> $-y \leq -1 \wedge$ $-y \leq -\frac{8}{3}x - 9$	$D = \{ x \mid -9 \leq x \leq -1 \}$ $W = \{ y \mid 0 \leq y \leq 8 \}$
Lösung	
<p>U1: $y \leq \frac{5}{7}x + 11\frac{1}{14}$</p> <p>U2: $y \leq -\frac{5}{7}x + 3\frac{13}{14}$</p> <p>U3: $y \geq -\frac{8}{3}x - 17\frac{2}{3}$</p> <p>U4: $y \geq 1$</p> <p>U5: $y \geq \frac{8}{3}x + 9$</p> <p>A $\left(-8\frac{1}{2} \mid 5 \right)$</p> <p>B $(-7 \mid 1)$</p> <p>C $(-3 \mid 1)$</p> <p>D $\left(-1\frac{1}{2} \mid 5 \right)$</p> <p>E $\left(-5 \mid 7\frac{1}{2} \right)$</p>	<p>The graph shows a coordinate system with x-axis from -9 to -1 and y-axis from 0 to 8. Five lines are plotted:</p> <ul style="list-style-type: none"> $g(x)$: red solid line, $y = \frac{5}{7}x + 11\frac{1}{14}$ $h(x)$: blue dashed line, $y = -\frac{5}{7}x + 3\frac{13}{14}$ $i(x)$: green dashed line, $y = -\frac{8}{3}x - 17\frac{2}{3}$ $j(x)$: magenta solid horizontal line, $y = 1$ $k(x)$: cyan solid line, $y = \frac{8}{3}x + 9$ <p>The feasible region is the intersection of the regions defined by these inequalities, which is a shaded area bounded by these lines.</p>

$-\frac{2}{3}x \leq 10\frac{1}{3} - y \wedge$ $\frac{2}{3}x \leq 3\frac{2}{3} - y \wedge$ 3 h) $y - 3\frac{1}{2}x \geq 11\frac{2}{3} \wedge$ $y + 21\frac{2}{3} \geq -3\frac{1}{2}x$	$D = \{ x \mid -8 \leq x \leq -2 \}$ $W = \{ y \mid -5 \leq y \leq 7 \}$
Lösung	
U1: $y \leq \frac{2}{3}x + 10\frac{1}{3}$ U2: $y \leq -\frac{2}{3}x + 3\frac{2}{3}$ U3: $y \geq 3\frac{1}{2}x + 11\frac{2}{3}$ U4: $y \geq -3\frac{1}{3}x - 21\frac{2}{3}$ A(-8 5) B(-5 7) C(-2 5) D(-5 5)	

$-\frac{2}{3}x \leq 3\frac{2}{3} - y \wedge$ $\frac{2}{3}x \leq 10\frac{1}{3} - y \wedge$ 3 i) $y - 3\frac{1}{3}x \geq -21\frac{2}{3} \wedge$ $y + 3\frac{1}{3}x \geq 11\frac{2}{3}$	$D = \{ x \mid 2 \leq x \leq 8 \}$ $W = \{ y \mid -5 \leq y \leq 7 \}$
Lösung	
$U1: y \leq \frac{2}{3}x + 3\frac{2}{3}$ $U2: y \leq -\frac{2}{3}x + 10\frac{1}{3}$ $U3: y \geq 3\frac{1}{3}x - 21\frac{2}{3}$ $U4: y \geq -3\frac{1}{3}x + 11\frac{2}{3}$ A (2 5) B (5 7) C (8 5) <u>D (5 -5)</u>	

$-\frac{y}{2} - 2 \geq \frac{3}{2}x \wedge$ $y - 2x \geq 1 \wedge$ <p>3 j) $-2y \leq x + 8 \wedge$</p> $-\frac{1}{11}y \geq -\frac{2}{11}x - 1 \wedge$ $3y - x \leq 8$	$D = \{ x \mid -6 \leq x \leq -1 \}$ $W = \{ y \mid -3 \leq y \leq 2 \}$
Lösung	
<p>U1: $y \leq -3x - 4$</p> <p>U2: $y \geq 2x + 1$</p> <p>U3: $y \geq -\frac{1}{2}x - 4$</p> <p>U4: $y \leq 2x + 11$</p> <p>U5: $y \leq \frac{1}{3}x + 2\frac{2}{3}$</p> <p>A (-5 1)</p> <p>B (-6 -1)</p> <p>C (-2 -3)</p> <p>D (-1 -1)</p> <p>E (-2 2)</p>	

$3y + x \leq 8 \wedge$ $-\frac{1}{11}y \geq \frac{2}{11}x - 1 \wedge$ <p>3 k) $y \geq \frac{1}{2}x - 4 \wedge$</p> $-2y \leq 4x - 2 \wedge$ $-\frac{1}{3}y \geq -x + 1 \frac{1}{3}$	$D = \{ x \mid 1 \leq x \leq 6 \}$ $W = \{ y \mid -3 \leq y \leq 2 \}$
Lösung	
<p>U1: $y \leq -\frac{1}{3}x + 2 \frac{2}{3}$</p> <p>U2: $y \leq -2x + 11$</p> <p>U3: $y \geq \frac{1}{2}x - 4$</p> <p>U4: $y \geq -2x + 1$</p> <p>U5: $y \leq 3x - 4$</p> <p>A(1 -1)</p> <p>B(2 -3)</p> <p>C(6 -1)</p> <p>D(5 1)</p> <p>E(2 2)</p>	<p>The graph shows five linear functions on a coordinate system with x-axis from 1 to 6 and y-axis from -3 to 2. The functions are: g(x) (red solid line), h(x) (blue dashed line), i(x) (green solid line), j(x) (magenta dashed line), and k(x) (cyan solid line). The lines are plotted within the domain D = [1, 6] and range W = [-3, 2].</p>