Lösungen Training Integration durch einfache Substitution I

Ergebnisse:

E1	Ergebnis
	$\int \frac{3}{4x+1}\,dx = \frac{3}{4}\cdot\ln(4x+1)+C$

E2	Ergebnis
	$\int_0^2 \frac{4}{4-x}\,dx = 4\cdot\ln(4)-4\cdot\ln(2) = 4\cdot\ln(2) \approx 2{,}773$

E3	Ergebnis
	$\int \frac{2}{(1-x)^2}\,dx = \frac{2}{1-x}+C$

E4	Ergebnis
	$\int \frac{6}{(2x-1)^3}\,dx = -\frac{3}{2}\cdot\frac{1}{(2x-1)^2}+C$

E5	Ergebnis
	$\int_{-2}^{2} \frac{10}{(x-4)^5}\,dx = -\frac{5}{2}\left[\frac{1}{16}-\frac{1}{1296}\right] = -\frac{25}{162} \approx -0{,}154$

E6	Ergebnis
	$\int_{-2}^{2} e^{1-x}\,dx = e^3 - e^{-1} \approx 19{,}718$

E7	Ergebnis
	$\int_0^4 e^{\frac{1}{2}x}\,dx = 2\cdot e^2 - 2\cdot e^0 = 2\cdot e^2 - 2 \approx 12{,}778$

E8	Ergebnis
	$\int_1^2 e^{4-2x}\,dx = \frac{1}{2}\cdot e^2 - \frac{1}{2}\cdot e^0 = \frac{1}{2}\cdot e^2 - \frac{1}{2} \approx 3{,}195$

E9	Ergebnis
	$\int_1^2 \frac{4}{e^{2x-4}}\,dx = 2\cdot e^2 - 2\cdot e^0 = 2\cdot e^2 - 2 \approx 12{,}788$

E10	Ergebnis
	$\int_0^2 \left(x-1-e^{-\frac{1}{2}x}\right)dx = -2\cdot e^0 - \left(-2\cdot e^{-1}\right) = -2+2\cdot e^{-1} \approx -1{,}264$

Ausführliche Lösungen:

A1	Ausführliche Lösung

$$\int \frac{3}{4x+1}\,dx \qquad \text{Substitution: } u = 4x+1 \qquad \frac{du}{dx} = 4 \Rightarrow dx = \frac{1}{4}du$$

$$\frac{3}{4}\int \frac{1}{u}\,du = \frac{3}{4}\cdot \ln(u) + C \qquad \text{mit } u = 4x+1 \text{ wird}$$

$$\int \frac{3}{4x+1}\,dx = \underline{\underline{\frac{3}{4}\cdot \ln(4x+1) + C}}$$

A2	Ausführliche Lösung

$$\int_0^2 \frac{4}{4-x}\,dx \qquad \text{Substitution: } u = 4-x$$

$$\frac{du}{dx} = -1 \Rightarrow dx = -du \quad \text{untere Grenze: } u(0) = 4 \quad \text{obere Grenze: } u(2) = 4-2 = 2$$

$$-4\int_4^2 \frac{1}{u}\,du = 4\int_2^4 \frac{1}{u}\,du = \left[4\cdot \ln(u)\right]_2^4 = 4\cdot \ln(4) - 4\cdot \ln(2)$$

$$= 4\left[\ln(4) - \ln(2)\right] = 4\cdot \ln\left(\frac{4}{2}\right) = 4\cdot \ln(2) \approx \underline{\underline{2{,}773}}$$

A3	Ausführliche Lösung

$$\int \frac{2}{(1-x)^2}\,dx \qquad \text{Substitution: } u = 1-x \qquad \frac{du}{dx} = -1 \Rightarrow dx = -du$$

$$-2\int \frac{1}{u^2}\,du = -2\int u^{-2}\,du = -2\cdot \frac{1}{-1}\cdot u^{-1} = \frac{2}{u} \qquad \text{mit } u = 1-x \text{ wird}$$

$$\int \frac{2}{(1-x)^2}\,dx = \underline{\underline{\frac{2}{1-x} + C}}$$

A4	Ausführliche Lösung

$$\int \frac{6}{(2x-1)^3}\,dx \qquad \text{Substitution: } u = 2x-1 \qquad \frac{du}{dx} = 2 \Rightarrow dx = \frac{1}{2}du$$

$$\frac{6}{2}\int \frac{1}{u^3}\,du = 3\int u^{-3}\,du = 3\cdot \frac{1}{-2}\cdot u^{-2} = -\frac{3}{2}\cdot \frac{1}{u^2} \qquad \text{mit } u = 2x-1 \text{ wird}$$

$$\int \frac{6}{(2x-1)^3}\,dx = \underline{\underline{-\frac{3}{2}\cdot \frac{1}{(2x-1)^2} + C}}$$

A5	Ausführliche Lösung

	$\int_{-2}^{2}\frac{10}{(x-4)^5}\,dx$ Substitution: $u=x-4$ $\frac{du}{dx}=1\Rightarrow dx=du$ untere Grenze: $u(-2)=-2-4=-6$ obere Grenze: $u(2)=2-4=-2$ $10\int_{-6}^{-2}\frac{1}{u^5}\,du=10\int_{-6}^{-2}u^{-5}\,du=\left[10\cdot\frac{1}{-4}\cdot u^{-4}\right]_{-6}^{-2}=-\frac{5}{2}\cdot\left[\frac{1}{u^4}\right]_{-6}^{-2}$ $=-\frac{5}{2}\left[\frac{1}{16}-\frac{1}{1296}\right]=\underline{\underline{-\frac{25}{162}\approx-0{,}154}}$

A6	Ausführliche Lösung
	$\int_{-2}^{2}e^{1-x}\,dx$ Substitution: $u=1-x$ $\frac{du}{dx}=-1\Rightarrow dx=-du$ untere Grenze: $u(-2)=1-(-2)=3$ obere Grenze: $u(2)=1-2=-1$ $-\int_{3}^{-1}e^{u}\,du=\int_{-1}^{3}e^{u}\,du=\left[e^{u}\right]_{-1}^{3}=\underline{\underline{e^3-e^{-1}\approx19{,}718}}$

A7	Ausführliche Lösung
	$\int_{0}^{4}e^{\frac{1}{2}x}\,dx$ Substitution: $u=\frac{1}{2}x$ $\frac{du}{dx}=\frac{1}{2}\Rightarrow dx=2du$ untere Grenze: $u(0)=0$ obere Grenze: $u(4)=2$ $2\int_{0}^{2}e^{u}\,du=2\left[e^{u}\right]_{0}^{2}=2\cdot e^2-2\cdot e^0=\underline{\underline{2\cdot e^2-2\approx12{,}778}}$

A8	Ausführliche Lösung
	$\int_{1}^{2}e^{4-2x}\,dx$ Substitution: $u=4-2x$ $\frac{du}{dx}=-2\Rightarrow dx=-\frac{1}{2}du$ untere Grenze: $u(1)=4-2=2$ obere Grenze: $u(2)=4-4=0$ $-\frac{1}{2}\int_{2}^{0}e^{u}\,du=\frac{1}{2}\int_{0}^{2}e^{u}\,du=\left[\frac{1}{2}e^{u}\right]_{0}^{2}=\frac{1}{2}\cdot e^2-\frac{1}{2}\cdot e^0=\underline{\underline{\frac{1}{2}\cdot e^2-\frac{1}{2}\approx3{,}195}}$

A9	Ausführliche Lösung
	$\int_{1}^{2}\frac{4}{e^{2x-4}}\,dx=4\int_{1}^{2}e^{4-2x}\,dx$ Substitution: $u=4-2x$ $\frac{du}{dx}=-2\Rightarrow dx=-\frac{1}{2}du$ untere Grenze: $u(1)=4-2=2$ obere Grenze: $u(2)=4-4=0$ $-4\cdot\frac{1}{2}\int_{2}^{0}e^{u}\,du=2\int_{0}^{2}e^{u}\,du=\left[2e^{u}\right]_{0}^{2}=2\cdot e^2-2\cdot e^0=\underline{\underline{2\cdot e^2-2\approx12{,}788}}$

A10	Ausführliche Lösung

	$\int_0^2 \left(x-1-e^{-\frac{1}{2}x} \right) dx = \int_0^2 (x-1)\,dx - \int_0^2 e^{-\frac{1}{2}x}\,dx$ 1. Integral: $\int_0^2 (x-1)\,dx = \left[\frac{1}{2}x^2 - x \right]_0^2 = \frac{1}{2}\cdot 4 - 2 - (0) = 0$ 2. Integral: $-\int_0^2 e^{-\frac{1}{2}x}\,dx$ Substitution: $u = -\frac{1}{2}x$ $\frac{du}{dx} = -\frac{1}{2} \Rightarrow dx = -2du$ untere Grenze: $u(0) = 0$ obere Grenze: $u(2) = -\frac{1}{2}\cdot 2 = -1$ $= (-1)(-2)\int_0^{-1} e^u\,du = 2\int_0^{-1} e^u\,du = -2\int_{-1}^{0} e^u\,du = \left[-2e^u \right]_{-1}^{0}$ $= -2\cdot e^0 - \left(-2\cdot e^{-1} \right) = \underline{\underline{-2 + 2\cdot e^{-1} \approx -1{,}264}}$