Lösungen Training Integration einfacher e- Funktionen I
Integrieren einfacher e- Funktionen

Ergebnisse:

E1	Ergebnis
	$F(x)=\int -e^{-x}\,dx = e^{-x}+C \quad \text{Probe}: F(x)=e^{-x}+C \Rightarrow F'(x)=-e^{-x}$

E2	Ergebnis
	$F(x)=\int \frac{1}{2}e^{2x}\,dx=\frac{1}{4}e^{2x}+C \quad \text{Probe}: F(x)=\frac{1}{4}e^{2x}+C \Rightarrow F'(x)=\frac{1}{2}e^{2x}$

E3	Ergebnis
	$F(x)=\int 2\cdot e^{-\frac{1}{4}x}\,dx=-8\cdot e^{-\frac{1}{4}x}+C \quad \text{Probe}: F(x)=-8\cdot e^{-\frac{1}{4}x}+C \Rightarrow F'(x)=2\cdot e^{-\frac{1}{4}x}$

E4	Ergebnis
	$F(x)=\int \frac{3}{4}\cdot e^{3x-4}\,dx=\frac{1}{4}\cdot e^{3x-4}+C \quad \text{Probe}: F(x)=\frac{1}{4}\cdot e^{3x-4}+C \Rightarrow F'(x)=\frac{3}{4}\cdot e^{3x-4}$

E5	Ergebnis
	$\int_0^2 e^{1-x}\,dx = e^1-e^{-1}=e-\frac{1}{e}\approx 2{,}350$

E6	Ergebnis
	$\int_{-1}^{2} e^{\frac{1}{2}x}\,dx = 2\cdot\left[e^1-e^{-\frac{1}{2}}\right]\approx 4{,}224$

E7	Ergebnis
	$\int_1^2 e^{4-2x}\,dx=\frac{1}{2}\cdot\left[e^2-e^0\right]=\frac{1}{2}\cdot\left[e^2-1\right]\approx 3{,}195$

E8	Ergebnis
	$\int_0^{\ln(2)} -\frac{1}{2}e^{-x}\,dx=-\frac{1}{2}\cdot\left[e^0-e^{-\ln(2)}\right]=-\frac{1}{2}\cdot\left[1-\frac{1}{e^{\ln(2)}}\right]=-\frac{1}{2}\cdot\left[1-\frac{1}{2}\right]=-\frac{1}{4}$

E9	Ergebnis
	$\int_1^2 \frac{4}{e^{2x-4}}\,dx=2\cdot\left[e^2-e^0\right]=2\cdot\left[e^2-1\right]\approx 12{,}778$

E10	Ergebnis
	$\int_0^4 -\frac{1}{2}\cdot e^{-\frac{1}{4}x}\,dx=-2\left[e^0-e^{-1}\right]\approx -1{,}264$

Ausführliche Lösungen:

A1	Ausführliche Lösung
	$F(x)=\int F'(x)\,dx=\int -e^{-x}\,dx=-\int e^{-x}\,dx$ $\text{Substitution}: u(x)=-x\Rightarrow \frac{du}{dx}=-1\Leftrightarrow dx=-du$ $-\int e^{-x}\,dx=-\int(-1)\cdot e^{u}\,du=\int e^{u}\,du=e^{u}+C$ $\Rightarrow F(x)=\int -e^{-x}\,dx=\underline{\underline{e^{-x}+C}}$ $\text{Probe}: F(x)=e^{-x}+C\Rightarrow F'(x)=-e^{-x}$

A2	Ausführliche Lösung
	$F(x)=\int F'(x)\,dx=\int \frac{1}{2}e^{2x}\,dx=\frac{1}{2}\int e^{2x}\,dx$ $\text{Substitution}: u(x)=2x\Rightarrow \frac{du}{dx}=2\Leftrightarrow dx=\frac{1}{2}du$ $\int \frac{1}{2}e^{2x}\,dx=\frac{1}{2}\int \frac{1}{2}e^{u}\,du=\frac{1}{4}\int e^{u}\,du=\frac{1}{4}e^{u}+C$ $\Rightarrow F(x)=\int \frac{1}{2}e^{2x}\,dx=\underline{\underline{\frac{1}{4}e^{2x}+C}}$ $\text{Probe}: F(x)=\frac{1}{4}e^{2x}+C\Rightarrow F'(x)=\frac{1}{2}e^{2x}$

A3	Ausführliche Lösung
	$F(x)=\int F'(x)\,dx=\int 2\cdot e^{-\frac{1}{4}x}\,dx=2\int e^{-\frac{1}{4}x}\,dx$ $\text{Substitution}: u(x)=-\frac{1}{4}x\Rightarrow \frac{du}{dx}=-\frac{1}{4}\Leftrightarrow dx=-4\cdot du$ $\int 2\cdot e^{-\frac{1}{4}x}\,dx=2\int(-4)\cdot e^{u}\,du=-8\int e^{u}\,du=-8\cdot e^{u}+C$ $\Rightarrow F(x)=\int 2\cdot e^{-\frac{1}{4}x}\,dx=\underline{\underline{-8\cdot e^{-\frac{1}{4}x}+C}}$ $\text{Probe}: F(x)=-8\cdot e^{-\frac{1}{4}x}+C\Rightarrow F'(x)=2\cdot e^{-\frac{1}{4}x}$

A4 Ausführliche Lösung

$$F(x) = \int F'(x)\,dx = \int \frac{3}{4}\cdot e^{3x-4}\,dx = \frac{3}{4}\int e^{3x-4}\,dx$$

$$\text{Substitution}: u(x) = 3x-4 \Rightarrow \frac{du}{dx} = 3 \Leftrightarrow dx = \frac{1}{3}\cdot du$$

$$\int \frac{3}{4}\cdot e^{3x-4}\,dx = \frac{3}{4}\int \frac{1}{3}\cdot e^{u}\,du = \frac{1}{4}\int e^{u}\,du = \frac{1}{4}\cdot e^{u} + C$$

$$\Rightarrow F(x) = \int \frac{3}{4}\cdot e^{3x-4}\,dx = \underline{\underline{\frac{1}{4}\cdot e^{3x-4} + C}}$$

$$\text{Probe}: F(x) = \frac{1}{4}\cdot e^{3x-4} + C \Rightarrow F'(x) = \frac{3}{4}\cdot e^{3x-4}$$

A5 Ausführliche Lösung

$$\int_0^2 e^{1-x}\,dx \qquad \text{Substitution: } u(x) = 1-x \Rightarrow \frac{du}{dx} = -1 \Leftrightarrow dx = -du$$

$$\text{untere Grenze: } u(0) = 1-0 = 1 \qquad \text{obere Grenze: } u(2) = 1-2 = -1$$

$$\int_0^2 e^{1-x}\,dx = \int_1^{-1} (-1)\cdot e^{u}\,du = -\int_1^{-1} e^{u}\,du = \int_{-1}^{1} e^{u}\,du = \left[e^{u}\right]_{-1}^{1} = e^{1} - e^{-1} = e - \frac{1}{e} \approx \underline{\underline{2{,}350}}$$

A6 Ausführliche Lösung

$$\int_{-1}^{2} e^{\frac{1}{2}x}\,dx \qquad \text{Substitution: } u(x) = \frac{1}{2}x \Rightarrow \frac{du}{dx} = \frac{1}{2} \Leftrightarrow dx = 2\cdot du$$

$$\text{untere Grenze: } u(-1) = \frac{1}{2}\cdot(-1) = -\frac{1}{2} \qquad \text{obere Grenze: } u(2) = \frac{1}{2}\cdot 2 = 1$$

$$\int_{-1}^{2} e^{\frac{1}{2}x}\,dx = 2\int_{-\frac{1}{2}}^{1} e^{u}\,du = 2\cdot\left[e^{u}\right]_{-\frac{1}{2}}^{1} = 2\cdot\left[e^{1} - e^{-\frac{1}{2}}\right] \approx \underline{\underline{4{,}224}}$$

A7 Ausführliche Lösung

$$\int_1^2 e^{4-2x}\,dx \qquad \text{Substitution: } u(x) = 4-2x \Rightarrow \frac{du}{dx} = -2 \Leftrightarrow dx = -\frac{1}{2}\cdot du$$

$$\text{untere Grenze: } u(1) = 4-2 = 2 \qquad \text{obere Grenze: } u(2) = 4-4 = 0$$

$$\int_1^2 e^{4-2x}\,dx = -\frac{1}{2}\int_2^0 e^{u}\,du = \frac{1}{2}\int_0^2 e^{u}\,du = \frac{1}{2}\cdot\left[e^{u}\right]_0^2 = \frac{1}{2}\cdot\left[e^{2} - e^{0}\right] = \frac{1}{2}\cdot\left[e^{2} - 1\right] \approx \underline{\underline{3{,}195}}$$

A8 Ausführliche Lösung

$$\int_0^{\ln(2)} -\frac{1}{2}e^{-x}\,dx = -\frac{1}{2}\int_0^{\ln(2)} e^{-x}\,dx \qquad \text{Substitution: } u(x) = -x \Rightarrow \frac{du}{dx} = -1 \Leftrightarrow dx = -du$$

untere Grenze: $u(0) = 0$ obere Grenze: $u(\ln(2)) = -\ln(2)$

$$-\frac{1}{2}\int_0^{\ln(2)} e^{-x}\,dx = -\frac{1}{2}\int_0^{-\ln(2)} (-1)\cdot e^{u}\,du = \frac{1}{2}\int_0^{-\ln(2)} e^{u}\,du = -\frac{1}{2}\int_{-\ln(2)}^{0} e^{u}\,du$$

$$= -\frac{1}{2}\cdot\left[e^{u}\right]_{-\ln(2)}^{0} = -\frac{1}{2}\cdot\left[e^{0} - e^{-\ln(2)}\right] = -\frac{1}{2}\cdot\left[1 - \frac{1}{e^{\ln(2)}}\right] = -\frac{1}{2}\cdot\left[1 - \frac{1}{2}\right] = \underline{\underline{-\frac{1}{4}}}$$

Bemerkung: $e^{\ln(a)} = a \Rightarrow e^{\ln(2)} = 2$

A9 Ausführliche Lösung

$$\int_1^2 \frac{4}{e^{2x-4}}\,dx = 4\int_1^2 e^{4-2x}\,dx \qquad \text{Substitution: } u(x) = 4 - 2x \Rightarrow \frac{du}{dx} = -2 \Leftrightarrow dx = -\frac{1}{2}du$$

untere Grenze: $u(1) = 4 - 2 = 2$ obere Grenze: $u(2) = 4 - 4 = 0$

$$4\int_1^2 e^{4-2x}\,dx = 4\int_2^0 \left(-\frac{1}{2}\right)\cdot e^{u}\,du = -2\int_2^0 e^{u}\,du = 2\int_0^2 e^{u}\,du$$

$$= 2\cdot\left[e^{u}\right]_0^2 = 2\cdot\left[e^{2} - e^{0}\right] = 2\cdot\left[e^{2} - 1\right] \approx \underline{\underline{12,778}}$$

A10 Ausführliche Lösung

$$\int_0^4 -\frac{1}{2}\cdot e^{-\frac{1}{4}x}\,dx = -\frac{1}{2}\int_0^4 e^{-\frac{1}{4}x}\,dx \qquad \text{Substitution: } u(x) = -\frac{1}{4}x \Rightarrow \frac{du}{dx} = -\frac{1}{4} \Leftrightarrow dx = -4\cdot du$$

untere Grenze: $u(0) = 0$ obere Grenze: $u(4) = -\frac{1}{4}\cdot 4 = -1$

$$-\frac{1}{2}\int_0^4 e^{-\frac{1}{4}x}\,dx = -\frac{1}{2}\int_0^{-1} (-4)\cdot e^{u}\,du = 2\int_0^{-1} e^{u}\,du = -2\int_{-1}^{0} e^{u}\,du$$

$$= -2\cdot\left[e^{u}\right]_{-1}^{0} = -2\left[e^{0} - e^{-1}\right] \approx \underline{\underline{-1,264}}$$