Lösungen Integration der e – Funktion I

Ergebnisse:

E1	Ergebnisse		
	a) $\int_0^1 e^x\,dx \approx 1{,}718$	b) $\int_{-1}^0 e^x\,dx \approx 0{,}632$	c) $\int_{-3}^0 e^x\,dx \approx 0{,}905$

E2	Ergebnisse		
	a) $\int_0^1 \left(e^x - 1\right) dx \approx 0{,}718$	b) $\int_0^3 \left(e^x - 1\right) dx \approx 16{,}086$	c) $\int_{-\infty}^0 e^x\,dx = 1$

E3	Ergebnisse		
	a) $\int_0^1 e^{-x}\,dx \approx 0{,}632$	b) $\int_{-1}^2 e^{-x}\,dx \approx 2{,}583$	c) $\int_0^\infty e^{-x}\,dx = 1$

E4	Ergebnisse		
	a) $\int_0^3 e^{-(x-3)}\,dx \approx 19{,}086$	b) $\int_1^3 e^{-(x-2)}\,dx \approx 2{,}350$	c) $\int_0^\infty e^{-(x-3)}\,dx = 1$

E5	Ergebnisse		
	a) $\int_0^3 \left(e^x - x + 1\right) dx \approx 17{,}586$	b) $\int_0^4 3 \cdot e^{x+2}\,dx \approx 1188{,}119$	c) $\int_0^5 \left(e^x + e^{-x}\right) dx \approx 148{,}406$

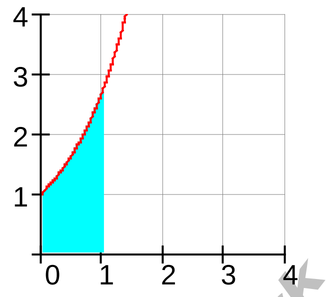
E6	Ergebnisse		
	a) $\int_0^2 k \cdot e^x\,dx = \frac{1}{2} \cdot e$ $\Leftrightarrow k \approx 0{,}213$	b) $\int_0^4 \left(e^x - k \cdot x\right) dx = 4$ $\Leftrightarrow k \approx 6{,}2$	c) $\int_0^k e^x\,dx = 2e$ $\Leftrightarrow k \approx 1{,}862$

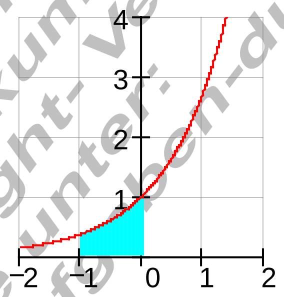
E7	Ergebnisse
	a) $\int_0^{\ln(2)} \left(e^{2x} - 2e^x\right) dx = \int_0^{\ln(2)} e^{2x}\,dx - 2 \cdot \int_0^{\ln(2)} e^x\,dx = -\frac{1}{2}$
	b) $\int_0^{\ln(k)} \left(e^{2x} - ke^x\right) dx = \int_0^{\ln(k)} e^{2x}\,dx - k \cdot \int_0^{\ln(k)} e^x\,dx = -\frac{1}{2}k^2 + k - \frac{1}{2}$
	c) $\int_0^4 \left(\frac{1}{4}e^x - 2 \cdot e^{\frac{1}{2}x}\right) dx = \frac{1}{4} \cdot \int_0^4 e^x\,dx - 2 \cdot \int_0^4 e^{\frac{1}{2}x}\,dx = \frac{1}{4}e^4 - 4e^2 + \frac{15}{4} \approx -12{,}175$

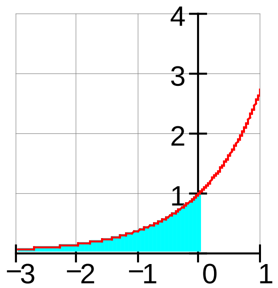
E8	Ergebnisse	
	a)	$\int_0^2 \left(e^x - 1\right) dx + \int_2^\infty f(2)\cdot e^{-(x-2)}\, dx = 2e^2 - 4 \approx 10{,}778$
	b)	$\int_0^3 \left(e^x - 1\right) dx + \int_3^\infty f(3)\cdot e^{-(x-3)}\, dx = 2e^3 - 5 \approx 35{,}171$

E9	Ergebnisse	
	a)	$\int_0^4 \left(e^x - 1\right) dx + \int_4^\infty f(4)\cdot e^{-(x-4)}\, dx = 2e^4 - 6 \approx 103{,}196$
	b)	$\int_0^k \left(e^x - 1\right) dx + \int_k^\infty f(k)\cdot e^{-(x-k)}\, dx = 2e^k - k - 2$

Ausführliche Lösungen:

A1	Ausführliche Lösung	
	a) $\int_0^1 e^x\,dx$ $= e^x \big|_0^1$ $= e^1 - e^0$ $= e - 1 \approx 1{,}718$	

A1	Ausführliche Lösung	
	b) $\int_{-1}^0 e^x\,dx$ $= e^x \big|_{-1}^0$ $= e^0 - e^{-1}$ $= 1 - e^{-1} \approx 0{,}632$	

A1	Ausführliche Lösung	
	c) $\int_{-3}^0 e^x\,dx$ $= e^x \big|_{-3}^0$ $= e^0 - e^{-3}$ $= 1 - e^{-3} \approx 0{,}950$	

A2 Ausführliche Lösung

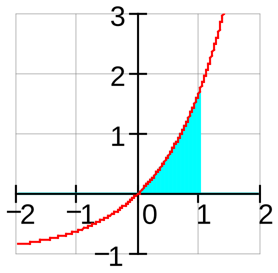
a)

$$\int_0^1 \left(e^x - 1\right)\,dx$$

$$= e^x - x \,\Big|_0^1$$

$$= e^1 - 1 - \left(e^0 - 0\right)$$

$$= e - 1 - 1 \approx 0{,}718$$



A2 Ausführliche Lösung

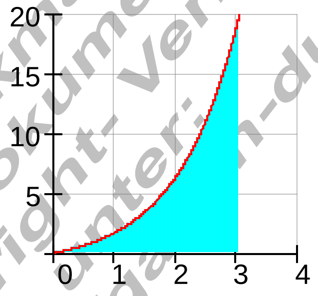
b)

$$\int_0^3 \left(e^x - 1\right)\,dx$$

$$= e^x - x \,\Big|_0^3$$

$$= e^3 - 3 - \left(e^0 - 0\right)$$

$$= e^3 - 3 - 1 \approx 16{,}086$$



A2 Ausführliche Lösung

c)

$$\int_{-\infty}^{0} e^x\,dx = \lim_{a \to -\infty} \int_a^0 e^x\,dx$$

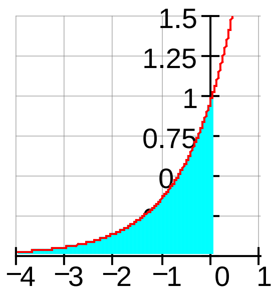
Berechnung zunächst ohne Grenzwertbildung

$$\int_a^0 e^x\,dx = e^x \,\Big|_a^0 = e^0 - e^a = 1 - e^a$$

Grenzwertbildung

$$\lim_{a \to -\infty} \int_a^0 e^x\,dx = \lim_{a \to -\infty} \left(1 - e^a\right)$$

$$= 1 - \underbrace{\lim_{a \to -\infty} e^a}_{0} = 1 \Rightarrow \int_{-\infty}^{0} e^x\,dx = 1$$



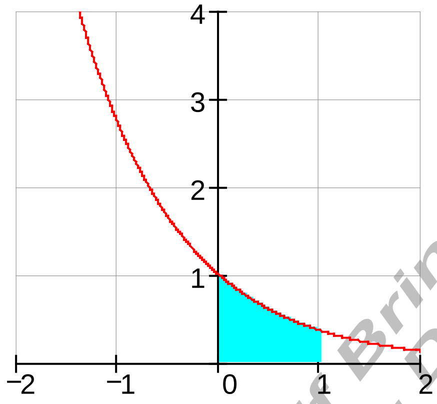
A3	Ausführliche Lösung

a) $\int_0^1 e^{-x}\,dx$ Lösung durch Substitution

Substitution: $u(x) = -x \Rightarrow u'(x) = \frac{du}{dx} = -1 \Leftrightarrow dx = -du$

untere Grenze: $u(0) = 0$ obere Grenze: $u(1) = -1$

$\Rightarrow -\int_0^{-1} e^u\,du = \int_{-1}^{0} e^u\,du = e^u\,\Big|_{-1}^{0} = e^0 - e^{-1} = 1 - e^{-1} \approx 0{,}632$



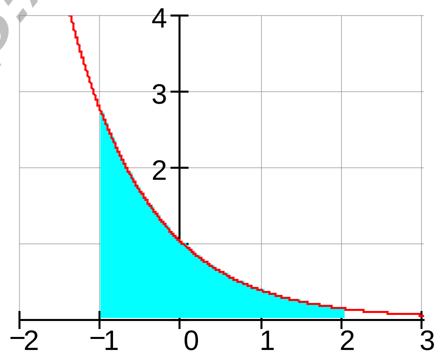
A3	Ausführliche Lösung

b) $\int_{-1}^{2} e^{-x}\,dx$ Lösung durch Substitution

Substitution: $u(x) = -x \Rightarrow u'(x) = \frac{du}{dx} = -1 \Leftrightarrow dx = -du$

untere Grenze: $u(-1) = -(-1) = 1$ obere Grenze: $u(2) = -2$

$\Rightarrow -\int_1^{-2} e^u\,du = \int_{-2}^{1} e^u\,du = e^u\,\Big|_{-2}^{1} = e^1 - e^{-2} = e - e^{-2} \approx 2{,}583$



A3 Ausführliche Lösung

c) $\int\limits_0^\infty e^{-x}\,dx = \lim\limits_{a\to\infty}\int\limits_0^a e^{-x}\,dx$ Lösung zunächst ohne Grenzwertbildung

$\int\limits_0^a e^{-x}\,dx$ Lösung durch Substitution

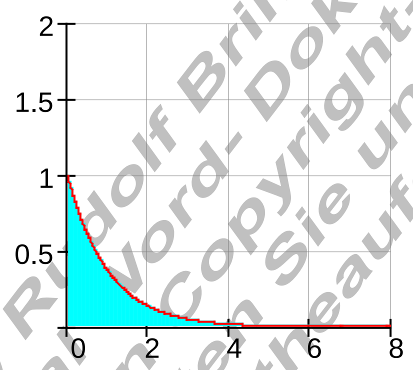
Substitution: $u(x) = -x \Rightarrow u'(x) = \frac{du}{dx} = -1 \Leftrightarrow dx = -du$

untere Grenze: $u(0) = -0 = 0$ obere Grenze: $u(a) = -a$

$$\Rightarrow -\int\limits_0^{-a} e^u\,du = \int\limits_{-a}^0 e^u\,du = e^u\,\Big|_{-a}^0 = e^0 - e^{-a} = 1 - e^{-a}$$

Die Grenzwertbildung;

$$\lim\limits_{a\to\infty}\int\limits_0^a e^{-x}\,dx = 1 - \underbrace{\lim\limits_{a\to\infty} e^{-a}}_{0} = 1 \Rightarrow \underline{\underline{\int\limits_0^\infty e^{-x}\,dx = 1}}$$



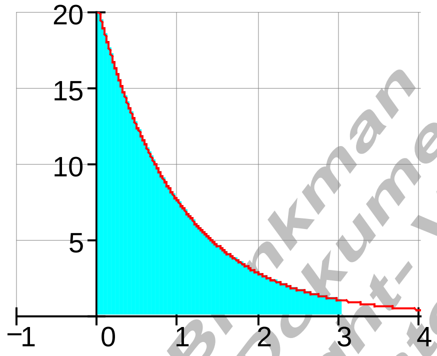
A4 Ausführliche Lösung

a) $\int_0^3 e^{-(x-3)}\,dx$ Lösung durch Substitution

Substitution: $u(x) = -(x-3) = -x+3 \Rightarrow u'(x) = \frac{du}{dx} = -1 \Leftrightarrow dx = -du$

untere Grenze: $u(0) = -(0-3) = 3$ obere Grenze: $u(3) = -(3-3) = 0$

$\Rightarrow -\int_3^0 e^u\,du = \int_0^3 e^u\,du = e^u\,\Big|_0^3 = e^3 - e^0 = e^3 - 1 \approx 19{,}086$



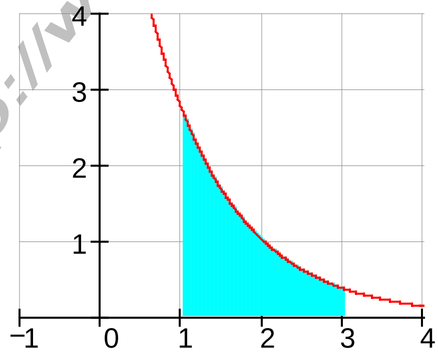
A4 Ausführliche Lösung

b) $\int_1^3 e^{-(x-2)}\,dx$ Lösung durch Substitution

Substitution: $u(x) = -(x-2) = -x+2 \Rightarrow u'(x) = \frac{du}{dx} = -1 \Leftrightarrow dx = -du$

untere Grenze: $u(1) = -(1-2) = 1$ obere Grenze: $u(3) = -(3-2) = -1$

$\Rightarrow -\int_1^{-1} e^u\,du = \int_{-1}^1 e^u\,du = e^u\,\Big|_{-1}^1 = e^1 - e^{-1} = e - e^{-1} \approx 2{,}350$



A4 Ausführliche Lösung

c) $\int_3^{\infty} e^{-(x-3)}\,dx = \lim_{a\to\infty}\int_3^{a} e^{-(x-3)}\,dx$ Lösung zunächst ohne Grenzwertbildung

$\int_3^{a} e^{-(x-3)}\,dx$ Lösung durch Substitution

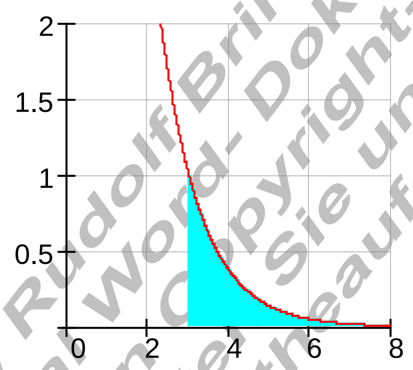
Substitution: $u(x) = -(x-3) = -x+3 \Rightarrow u'(x) = \frac{du}{dx} = -1 \Leftrightarrow dx = -du$

untere Grenze: $u(3) = -(3-3) = 0$ obere Grenze: $u(a) = -a+3$

$$\Rightarrow -\int_0^{-a+3} e^u\,du = \int_{-a+3}^{0} e^u\,du = e^u\Big|_{-a+3}^{0} = e^0 - e^{-a+3} = 1 - e^{-a+3}$$

Die Grenzwertbildung;

$$\lim_{a\to\infty}\int_0^{a} e^{-(x-3)}\,dx = 1 - \underbrace{\lim_{a\to\infty} e^{-a+3}}_{0} = 1 \Rightarrow \underline{\underline{\int_0^{\infty} e^{-(x-3)}\,dx = 1}}$$



A5 Ausführliche Lösung

a) $\int_0^3 \left(e^x - x + 1\right) dx = e^x - \frac{1}{2}x^2 + x\Big|_0^3$

$$= e^3 - \frac{1}{2}\cdot 9 + 3 - \left(e^0 - \frac{1}{2}\cdot 0 + 0\right)$$

$$= e^3 - \frac{9}{2} + 3 - 1 = e^3 - \frac{5}{2} \approx 17{,}586$$

A5	Ausführliche Lösung
	b) $\int_0^4 3\cdot e^{x+2}\,dx = 3\cdot\int_0^4 e^{x+2}\,dx$ Lösung durch Substitution Substitution: $u(x) = x+2 \Rightarrow u'(x) = \frac{du}{dx} = 1 \Leftrightarrow dx = du$ untere Grenze: $u(0) = 0+2 = 2$ obere Grenze: $u(4) = 4+2 = 6$ $\Rightarrow 3\cdot\int_2^6 e^u\,du = 3\cdot\int_2^6 e^u\,du = 3\cdot e^u\,\big\vert_2^6 = 3\cdot\left(e^6 - e^2\right) \approx 1188{,}119$

A5	Ausführliche Lösung
	c) $\int_0^5 \left(e^x + e^{-x}\right)dx = \underbrace{\int_0^5 e^x\,dx}_{I} + \underbrace{\int_0^5 e^{-x}\,dx}_{II} = I + II$ $I: \int_0^5 e^x\,dx = e^x\,\big\vert_0^5 = e^5 - e^0 = e^5 - 1$ $II: \int_0^5 e^{-x}\,dx =$ Lösung durch Substitution Substitution: $u(x) = -x \Rightarrow u'(x) = \frac{du}{dx} = -1 \Leftrightarrow dx = -du$ untere Grenze: $u(0) = 0$ obere Grenze: $u(5) = -5$ $\Rightarrow -\int_0^{-5} e^u\,du = \int_{-5}^0 e^u\,du = e^u\,\big\vert_{-5}^0 = e^0 - e^{-5} = 1 - e^{-5}$ $\int_0^5 \left(e^x + e^{-x}\right)dx = I + II = e^5 - 1 + 1 - e^{-5} = e^5 - e^{-5} \approx 148{,}406$

A6	Ausführliche Lösung
	a) $\int_0^2 k\cdot e^x\,dx = \frac{1}{2}\cdot e$ $\int_0^2 k\cdot e^x\,dx = k\cdot\int_0^2 e^x\,dx = k\cdot e^x\,\big\vert_0^2 = k\cdot\left(e^2 - e^0\right) = k\cdot\left(e^2 - 1\right)$ gleichsetzen: $k\cdot\left(e^2-1\right) = \frac{1}{2}\cdot e \quad \vert : \left(e^2-1\right)$ $\Leftrightarrow k = \frac{\frac{1}{2}\cdot e}{\left(e^2-1\right)} = \frac{1}{2}\cdot\frac{e}{e^2-1} \approx 0{,}213$

A6	Ausführliche Lösung
b)	$\int_0^4 \left(e^x - k\cdot x\right)dx = 4$ $\int_0^4 \left(e^x - k\cdot x\right)dx = e^x - \frac{1}{2}kx^2 \Big\|_0^4 = e^4 - \frac{1}{2}k\cdot 16 - \left(e^0 - 0\right) = e^4 - 8k - 1$ gleichsetzen: $e^4 - 8k - 1 = 4 \mid -e^4 + 1$ $\Leftrightarrow -8k = -e^4 + 5 \mid :(-8)$ $\Leftrightarrow k = \frac{-e^4+5}{-8} = \frac{1}{8}e^4 - \frac{5}{8} \approx 6{,}2$

A6	Ausführliche Lösung
c)	$\int_0^k e^x\,dx = 2e$ $\int_0^k e^x\,dx = e^x \Big\|_0^k = e^k - e^0 = e^k - 1$ gleichsetzen: $e^k - 1 = 2e \mid +1$ $\Leftrightarrow e^k = 2e+1 \mid \ln(\ )$ $\Leftrightarrow \ln\left(e^k\right) = \ln(2e+1)$ $\Leftrightarrow k\cdot \underbrace{\ln(e)}_{1} = \ln(2e+1)$ $\Leftrightarrow k = \ln(2e+1) \approx 1{,}862$

A7	Ausführliche Lösung
a)	$\int_0^{\ln(2)} \left(e^{2x} - 2e^x\right)dx = \underbrace{\int_0^{\ln(2)} e^{2x}\,dx}_{I} - 2\cdot\underbrace{\int_0^{\ln(2)} e^x\,dx}_{II} = I - 2\cdot II$ $I: \int_0^{\ln(2)} e^{2x}\,dx =$ Lösung durch Substitution Substitution: $u(x) = 2x \Rightarrow u'(x) = \frac{du}{dx} = 2 \Leftrightarrow dx = \frac{1}{2}du$ untere Grenze: $u(0) = 2\cdot 0 = 0$ obere Grenze: $u(\ln(2)) = 2\cdot\ln(2)$ $\Rightarrow \frac{1}{2}\int_0^{2\cdot\ln(2)} e^u\,du = \frac{1}{2}e^u \Big\|_0^{2\cdot\ln(2)} = \frac{1}{2}\left(e^{2\cdot\ln(2)} - e^0\right) = \frac{1}{2}\left[\left(\underbrace{e^{\ln(2)}}_{2}\right)^2 - 1\right] = \frac{1}{2}\left(2^2 - 1\right) = \frac{3}{2}$ $II: \int_0^{\ln(2)} e^x\,dx = e^u \Big\|_0^{2\cdot\ln(2)} = \underbrace{e^{\ln(2)}}_{2} - e^0 = 2 - 1 = 1$ $\int_0^{\ln(2)} \left(e^{2x} - 2e^x\right)dx = I - 2\cdot II = \frac{3}{2} - 2\cdot 1 = -\frac{1}{2}$

A7	Ausführliche Lösung

b) $\int_0^{\ln(k)} \left(e^{2x} - ke^x\right) dx = \underbrace{\int_0^{\ln(k)} e^{2x}\, dx}_{I} - k \cdot \underbrace{\int_0^{\ln(2)} e^x\, dx}_{II} = I - k \cdot II$

$I: \int_0^{\ln(k)} e^{2x}\, dx =$ Lösung durch Substitution

Substitution: $u(x) = 2x \Rightarrow u'(x) = \frac{du}{dx} = 2 \Leftrightarrow dx = \frac{1}{2}du$

untere Grenze: $u(0) = 2 \cdot 0 = 0$ obere Grenze: $u(\ln(k)) = 2 \cdot \ln(k)$

$\Rightarrow \frac{1}{2}\int_0^{2\cdot\ln(k)} e^u\, du = \frac{1}{2}e^u \Big|_0^{2\cdot\ln(k)} = \frac{1}{2}\left(e^{2\cdot\ln(k)} - e^0\right) = \frac{1}{2}\left[\left(\underbrace{e^{\ln(k)}}_{k}\right)^2 - 1\right] = \frac{1}{2}\left(k^2 - 1\right)$

$II: \int_0^{\ln(k)} e^x\, dx = e^u \Big|_0^{2\cdot\ln(k)} = \underbrace{e^{\ln(k)}}_{k} - e^0 = k - 1$

$\int_0^{\ln(k)} \left(e^{2x} - ke^x\right) dx = I - k \cdot II = \frac{1}{2}\left(k^2 - 1\right) - k \cdot (k - 1) = -\frac{1}{2}k^2 + k - \frac{1}{2}$

A7	Ausführliche Lösung

c) $\int_0^4 \left(\frac{1}{4}e^x - 2 \cdot e^{\frac{1}{2}x}\right) dx = \frac{1}{4} \cdot \underbrace{\int_0^4 e^x\, dx}_{I} - 2 \cdot \underbrace{\int_0^4 e^{\frac{1}{2}x}\, dx}_{II} = \frac{1}{4} \cdot I - 2 \cdot II$

$I: \int_0^4 e^x\, dx = e^x \Big|_0^4 = e^4 - e^0 = e^4 - 1$

$II: \int_0^4 e^{\frac{1}{2}x}\, dx =$ Lösung durch Substitution

Substitution: $u(x) = \frac{1}{2}x \Rightarrow u'(x) = \frac{du}{dx} = \frac{1}{2} \Leftrightarrow dx = 2du$

untere Grenze: $u(0) = \frac{1}{2} \cdot 0 = 0$ obere Grenze: $u(4) = \frac{1}{2} \cdot 4 = 2$

$\Rightarrow 2 \cdot \int_0^{2} e^u\, du = 2 \cdot e^u \Big|_0^2 = 2\left(e^2 - e^0\right) = 2e^2 - 2$

$\int_0^4 \left(\frac{1}{4}e^x - 2 \cdot e^{\frac{1}{2}x}\right) dx = \frac{1}{4} \cdot I - 2 \cdot II = \frac{1}{4} \cdot \left(e^4 - 1\right) - 2 \cdot \left(2e^2 - 2\right)$

$= \frac{1}{4}e^4 - \frac{1}{4} - 4e^2 + 4 = \frac{1}{4}e^4 - 4e^2 + \frac{15}{4} \approx -12{,}157$

A8 Ausführliche Lösung

a)

$f(x) = e^x - 1 \qquad g(x) = f(2)\cdot e^{-(x-2)}$ mit $f(2) = e^2 - 1$

$$A = \int_0^2 f(x)\,dx + \int_2^\infty g(x)\,dx = \underbrace{\int_0^2 \left(e^x - 1\right)dx}_{I} + \left(e^2 - 1\right)\underbrace{\int_2^\infty e^{-(x-2)}\,dx}_{II} = I + \left(e^2 - 1\right)\cdot II$$

$$I: \int_0^2 \left(e^x - 1\right)dx = e^x - x\,\Big|_0^2 = e^2 - 2 - \left(e^0 - 0\right) = e^2 - 3$$

$$II: \int_2^\infty e^{-(x-2)}\,dx = \lim_{a\to\infty}\int_2^a e^{-(x-2)}\,dx$$ Lösung zunächst ohne Grenzwertbildung

$$\int_2^a e^{-(x-2)}\,dx$$ Lösung durch Substitution

Substitution: $u(x) = -(x-2) = -x + 2 \Rightarrow u'(x) = \frac{du}{dx} = -1 \Leftrightarrow dx = -du$

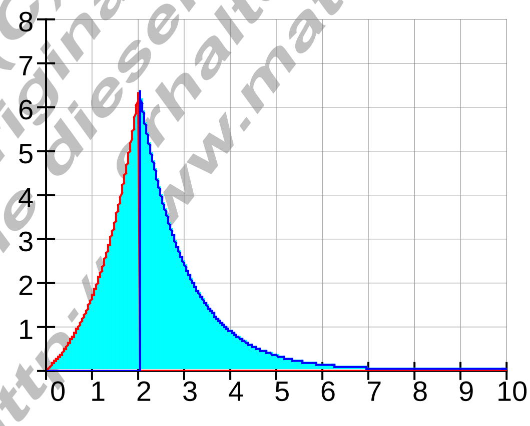
untere Grenze: $u(2) = -(2-2) = 0$ obere Grenze: $u(a) = -a + 2$

$$\Rightarrow -\int_0^{-a+2} e^u\,du = \int_{-a+2}^0 e^u\,du = e^u\,\Big|_{-a+2}^0 = e^0 - e^{-a+2} = 1 - e^{-a+2}$$

Die Grenzwertbildung;

$$\lim_{a\to\infty}\int_0^a e^{-(x-2)}\,dx = 1 - \underbrace{\lim_{a\to\infty} e^{-a+2}}_{0} = 1 \Rightarrow \int_2^\infty e^{-(x-2)}\,dx = 1$$

$$A = I + \left(e^2 - 1\right)\cdot II = e^2 - 3 + \left(e^2 - 1\right)\cdot 1 = e^2 - 3 + e^2 - 1 = 2e^2 - 4 \approx 10{,}778$$



A8 Ausführliche Lösung

b) $f(x) = e^x - 1 \qquad g(x) = f(3)\cdot e^{-(x-3)}$ mit $f(3) = e^3 - 1$

$$A = \int_0^3 f(x)\,dx + \int_3^\infty g(x)\,dx = \underbrace{\int_0^3 \left(e^x - 1\right) dx}_{I} + \left(e^3 - 1\right)\underbrace{\int_3^\infty e^{-(x-3)}\,dx}_{II} = I + \left(e^3 - 1\right)\cdot II$$

$$I: \int_0^3 \left(e^x - 1\right) dx = e^x - x \,\Big|_0^3 = e^3 - 3 - \left(e^0 - 0\right) = e^3 - 4$$

$$II: \int_3^\infty e^{-(x-3)}\,dx = \lim_{a\to\infty}\int_3^a e^{-(x-3)}\,dx$$ Lösung zunächst ohne Grenzwertbildung

$$\int_3^a e^{-(x-3)}\,dx$$ Lösung durch Substitution

Substitution: $u(x) = -(x-3) = -x + 3 \Rightarrow u'(x) = \frac{du}{dx} = -1 \Leftrightarrow dx = -du$

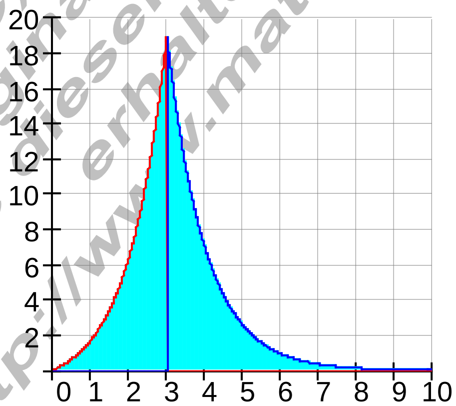
untere Grenze: $u(3) = -(3-3) = 0$ obere Grenze: $u(a) = -a + 3$

$$\Rightarrow -\int_0^{-a+3} e^u\,du = \int_{-a+3}^0 e^u\,du = e^u \,\Big|_{-a+3}^0 = e^0 - e^{-a+3} = 1 - e^{-a+3}$$

Die Grenzwertbildung;

$$\lim_{a\to\infty}\int_0^a e^{-(x-3)}\,dx = 1 - \underbrace{\lim_{a\to\infty} e^{-a+3}}_{0} = 1 \Rightarrow \int_3^\infty e^{-(x-3)}\,dx = 1$$

$$A = I + \left(e^3 - 1\right)\cdot II = e^3 - 4 + \left(e^3 - 1\right)\cdot 1 = e^3 - 4 + e^3 - 1 = 2e^3 - 5 \approx 35{,}171$$



A9 Ausführliche Lösung

a)

$$f(x)=e^x-1 \qquad g(x)=f(4)\cdot e^{-(x-4)} \text{ mit } f(4)=e^4-1$$

$$A=\int_0^4 f(x)\,dx+\int_4^\infty g(x)\,dx=\underbrace{\int_0^4\left(e^x-1\right)dx}_{I}+\left(e^4-1\right)\underbrace{\int_4^\infty e^{-(x-4)}\,dx}_{II}=I+\left(e^4-1\right)\cdot II$$

$$I:\int_0^4\left(e^x-1\right)dx=e^x-x\,\Big|_0^4=e^4-4-\left(e^0-0\right)=e^4-5$$

$$II:\int_4^\infty e^{-(x-4)}\,dx=\lim_{a\to\infty}\int_4^a e^{-(x-4)}\,dx \text{ Lösung zunächst ohne Grenzwertbildung}$$

$$\int_4^a e^{-(x-4)}\,dx \text{ Lösung durch Substitution}$$

Substitution: $u(x)=-(x-4)=-x+4\Rightarrow u'(x)=\frac{du}{dx}=-1\Leftrightarrow dx=-du$

untere Grenze: $u(4)=-(4-4)=0$ obere Grenze: $u(a)=-a+4$

$$\Rightarrow-\int_0^{-a+4}e^u\,du=\int_{-a+4}^0 e^u\,du=e^u\,\Big|_{-a+4}^0=e^0-e^{-a+4}=1-e^{-a+4}$$

Die Grenzwertbildung;

$$\lim_{a\to\infty}\int_0^a e^{-(x-4)}\,dx=1-\underbrace{\lim_{a\to\infty}e^{-a+4}}_{0}=1\Rightarrow\int_4^\infty e^{-(x-4)}\,dx=1$$

$$A=I+\left(e^4-1\right)\cdot II=e^4-5+\left(e^4-1\right)\cdot 1=e^4-5+e^4-1=2e^4-6\approx 103{,}196$$

A9	Ausführliche Lösung
	b) $f(x)=e^x-1 \qquad g(x)=f(k)\cdot e^{-(x-k)}$ mit $f(k)=e^k-1$ $A=\int_0^k f(x)\,dx+\int_k^\infty g(x)\,dx=\underbrace{\int_0^k (e^x-1)\,dx}_{I}+(e^k-1)\underbrace{\int_k^\infty e^{-(x-k)}\,dx}_{II}=I+(e^k-1)\cdot II$ $I:\int_0^k (e^x-1)\,dx=e^x-x\,\Big\vert_0^k=e^k-k-(e^0-0)=e^k-k-1$ $II:\int_k^\infty e^{-(x-k)}\,dx=\lim\limits_{a\to\infty}\int_k^a e^{-(x-k)}\,dx$ Lösung zunächst ohne Grenzwertbildung $\int_k^a e^{-(x-k)}\,dx$ Lösung durch Substitution Substitution: $u(x)=-(x-k)=-x+k\Rightarrow u'(x)=\frac{du}{dx}=-1\Leftrightarrow dx=-du$ untere Grenze: $u(k)=-(k-k)=0$ obere Grenze: $u(a)=-a+k$ $\Rightarrow -\int_0^{-a+k} e^u\,du=\int_{-a+k}^0 e^u\,du=e^u\,\Big\vert_{-a+k}^0=e^0-e^{-a+k}=1-e^{-a+k}$ Die Grenzwertbildung; $\lim\limits_{a\to\infty}\int_0^a e^{-(x-k)}\,dx=1-\underbrace{\lim\limits_{a\to\infty} e^{-a+k}}_{0}=1\Rightarrow\int_k^\infty e^{-(x-k)}\,dx=1$ $A=I+(e^k-1)\cdot II=e^k-k-1+(e^k-1)\cdot 1=e^k-k-1+e^k-1=2e^k-k-2$