Lösungen Training Differentialrechnung III

Ergebnisse:

E1	Ergebnis:
	$f(x) = x^3 - 6x^2 + 9x \qquad x_0 = 2 \Rightarrow P_0(2 \mid 2) \qquad t(x) = -3x + 8$

E2	Ergebnis:
	$f(x) = \frac{1}{4}x^3 - \frac{9}{4}x^2 + \frac{15}{4}x + \frac{9}{4} \qquad x_0 = 2 \Rightarrow P_0\left(2 \,\middle|\, \frac{11}{4} = 2{,}75\right) \qquad t(x) = -\frac{9}{4}x + \frac{29}{4}$

E3	Ergebnis:
	$f(x) = \frac{1}{2}x^3 - \frac{1}{2}x^2 - 4x + 4 \qquad x_0 = -1 \Rightarrow P_0(-1 \mid 7) \qquad t(x) = -\frac{3}{2}x + \frac{11}{2}$

E4	Ergebnis:
	$f(x) = x^3 + 3x^2 - 2 \qquad x_0 = 1 \Rightarrow P_0(1 \mid 2) \qquad t(x) = 9x - 7$

E5	Ergebnis:
	$f(x) = \frac{1}{2}x^3 - \frac{1}{2}x^2 - \frac{5}{2}x + 3 \qquad x_0 = 1 \Rightarrow P_0\left(1 \,\middle|\, \frac{1}{2} = 0{,}5\right) \qquad t(x) = -2x + \frac{5}{2}$

E6	Ergebnis:
	$f(x) = x^3 - x^2 - 5x - 2 \qquad x_0 = -\frac{3}{2} \Rightarrow P_0\left(-\frac{3}{2} \,\middle|\, -\frac{1}{8}\right) \qquad t(x) = \frac{19}{4}x + 7$

E7	Ergebnis:
	$f(x) = \frac{1}{4}x^3 - \frac{5}{4}x^2 + \frac{1}{2}x + 2 \qquad x_0 = 3 \Rightarrow P_0(3 \mid -1) \qquad t(x) = -\frac{1}{4}x - \frac{1}{4}$

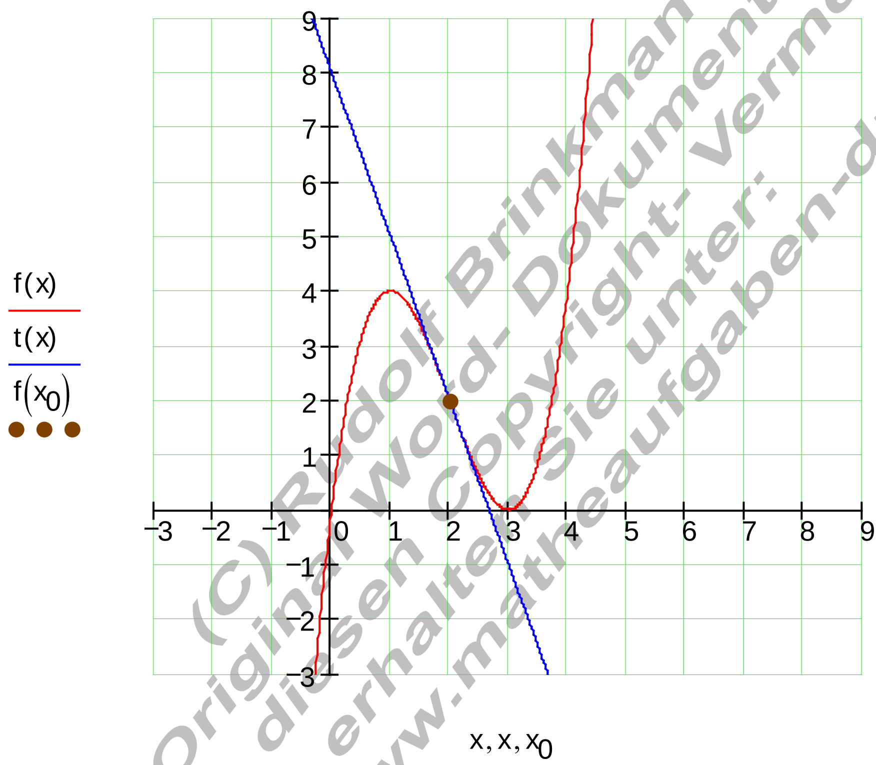
E8	Ergebnis:
	$f(x) = x^3 - \frac{3}{2}x^2 - 6x + 2 \qquad x_0 = 1 \Rightarrow P_0\left(1 \,\middle|\, -\frac{9}{2}\right) \qquad t(x) = -6x + \frac{3}{2}$

E9	Ergebnis:
	$f(x) = \frac{1}{2}x^3 - \frac{1}{2}x^2 - 4x + 4 \qquad x_0 = 0 \Rightarrow P_0(0 \mid 4) \qquad t(x) = -4x + 4$

E10	Ergebnis:
	$f(x) = \frac{1}{3}x^3 - \frac{1}{3}x^2 - 4x + 4 \qquad x_0 = 1 \Rightarrow P_0(1 \mid 0) \qquad t(x) = -\frac{11}{3}x + \frac{11}{3}$

Ausführliche Lösungen:

A1	Ausführliche Lösung (Berechnungen)
	$f(x) = x^3 - 6x^2 + 9x \qquad x_0 = 2 \qquad t(x) = f'(x_0)(x - x_0) + f(x_0)$ $f'(x) = 3x^2 - 12x + 9 \Rightarrow f'(x_0) = f'(2) = 12 - 24 + 9 = -3$ $f(x_0) = f(2) = 8 - 24 + 18 = 2 \Rightarrow \underline{\underline{P_0(2 \mid 2)}}$ $t(x) = -3(x-2) + 2 = -3x + 6 + 2 = \underline{\underline{-3x + 8}}$

A1	Ausführliche Lösung (Die Graphen)
	$f(x) = x^3 - 6x^2 + 9x \qquad x_0 = 2 \Rightarrow P_0(2 \mid 2) \qquad t(x) = -3x + 8$
	

A2 Ausführliche Lösung (Berechnungen)

$$f(x)=\frac{1}{4}x^3-\frac{9}{4}x^2+\frac{15}{4}x+\frac{9}{4} \qquad x_0=2 \qquad t(x)=f'(x_0)(x-x_0)+f(x_0)$$

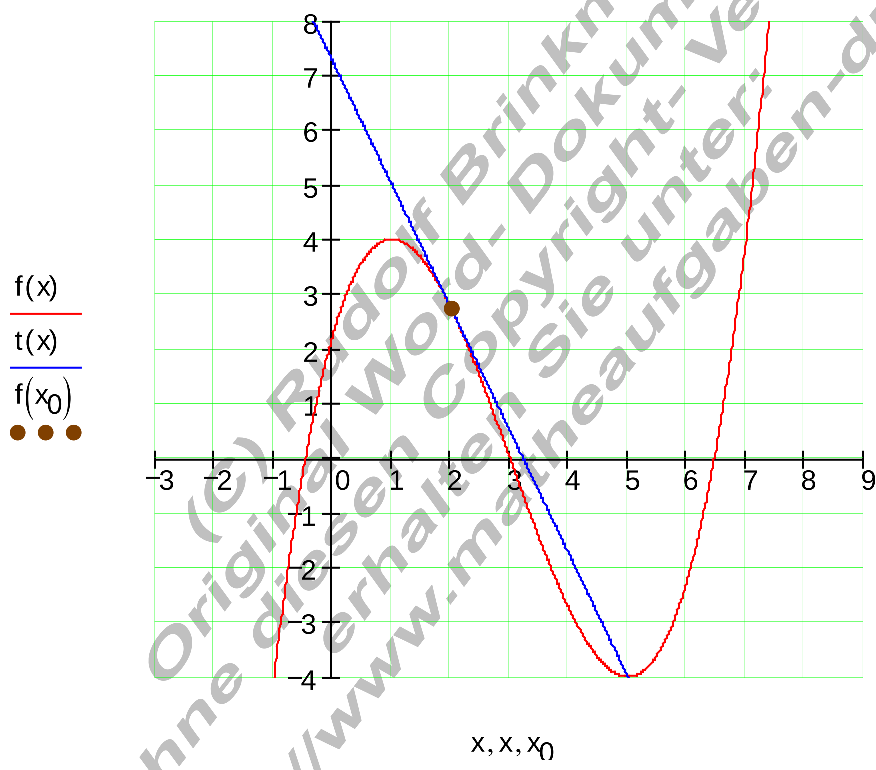
$$f'(x)=\frac{3}{4}x^2-\frac{9}{2}x+\frac{15}{4} \Rightarrow f'(x_0)=f'(2)=\frac{3}{4}\cdot 4-\frac{9}{2}\cdot 2+\frac{15}{4}=\frac{12}{4}-\frac{36}{4}+\frac{15}{4}=-\frac{9}{4}$$

$$f(x_0)=f(2)=\frac{1}{4}\cdot 8-\frac{9}{4}\cdot 4+\frac{15}{4}\cdot 2+\frac{9}{4}=\frac{8}{4}-\frac{36}{4}+\frac{30}{4}+\frac{9}{4}=\frac{11}{4} \Rightarrow \underline{\underline{P_0\left(2 \,\middle|\, \frac{11}{4}=2{,}75\right)}}$$

$$t(x)=-\frac{9}{4}(x-2)+\frac{11}{4}=-\frac{9}{4}x+\frac{18}{4}+\frac{11}{4}=\underline{\underline{-\frac{9}{4}x+\frac{29}{4}}}$$

A2 Ausführliche Lösung (Die Graphen)

$$f(x)=\frac{1}{4}x^3-\frac{9}{4}x^2+\frac{15}{4}x+\frac{9}{4} \qquad x_0=2 \Rightarrow P_0\left(2 \,\middle|\, \frac{11}{4}=2{,}75\right) \qquad t(x)=-\frac{9}{4}x+\frac{29}{4}$$



A3 Ausführliche Lösung (Berechnungen)

$$f(x)=\frac{1}{2}x^3-\frac{1}{2}x^2-4x+4 \qquad x_0=-1 \qquad t(x)=f'(x_0)(x-x_0)+f(x_0)$$

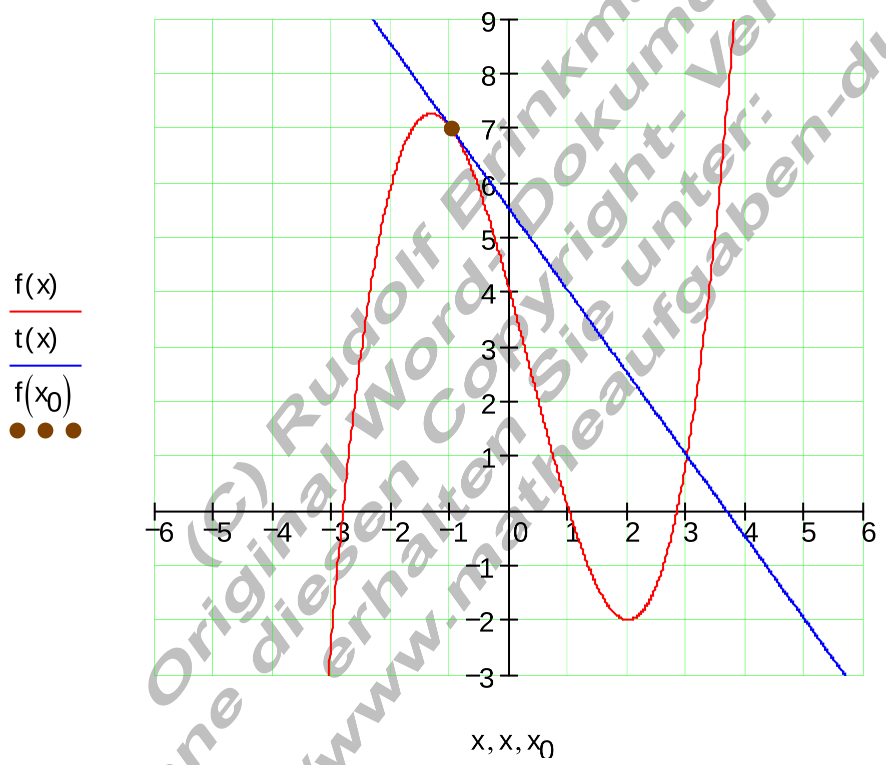
$$f'(x)=\frac{3}{2}x^2-x-4 \Rightarrow f'(x_0)=f'(-1)=\frac{3}{2}+1-4=\frac{3}{2}+\frac{2}{2}-\frac{8}{2}=-\frac{3}{2}$$

$$f(x_0)=f(-1)=\frac{1}{2}\cdot(-1)-\frac{1}{2}\cdot 1-4\cdot(-1)+4=-\frac{1}{2}-\frac{1}{2}+\frac{8}{2}+\frac{8}{2}=7 \Rightarrow \underline{\underline{P_0(-1|7)}}$$

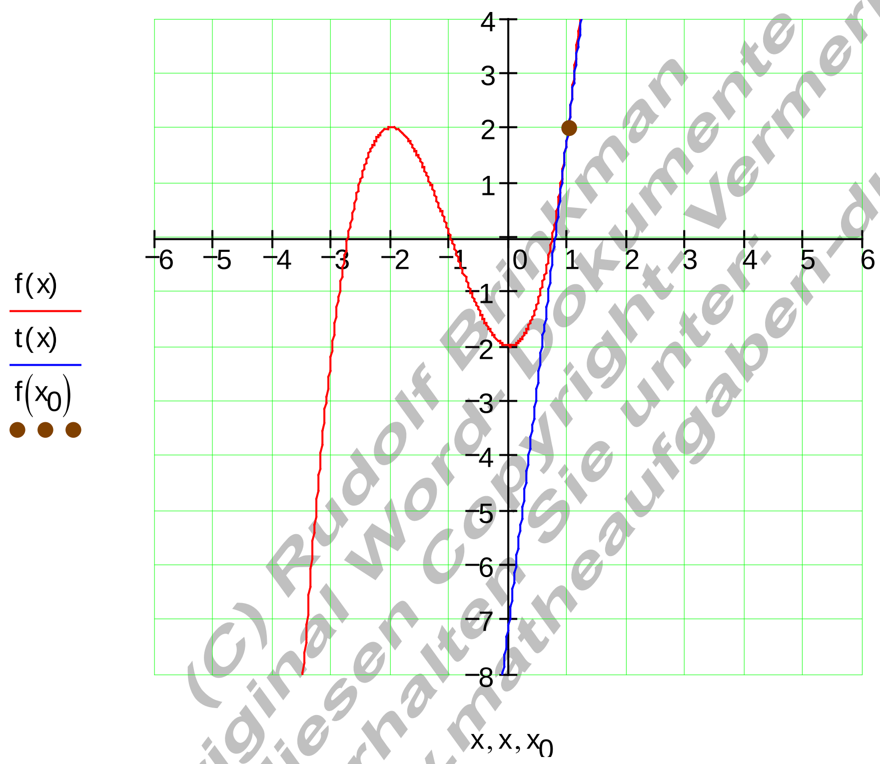
$$t(x)=-\frac{3}{2}(x+1)+7=-\frac{3}{2}x-\frac{3}{2}+\frac{14}{2}=\underline{\underline{-\frac{3}{2}x+\frac{11}{2}}}$$

A3 Ausführliche Lösung (Die Graphen)

$$f(x)=\frac{1}{2}x^3-\frac{1}{2}x^2-4x+4 \qquad x_0=-1 \Rightarrow P_0(-1|7) \qquad t(x)=-\frac{3}{2}x+\frac{11}{2}$$



A4	Ausführliche Lösung (Berechnungen)
	$f(x) = x^3 + 3x^2 - 2 \quad x_0 = 1 \quad t(x) = f'(x_0)(x - x_0) + f(x_0)$
	$f'(x) = 3x^2 + 6x \Rightarrow f'(x_0) = f'(1) = 3 + 6 = 9$
	$f(x_0) = f(1) = 1^3 + 3 \cdot 1^2 - 2 = 1 + 3 - 2 = 2 \Rightarrow \underline{\underline{P_0(1 \mid 2)}}$
	$t(x) = 9(x-1) + 2 = 9x - 9 + 2 = \underline{\underline{9x - 7}}$

A4	Ausführliche Lösung (Die Graphen)
	$f(x) = x^3 + 3x^2 - 2 \quad x_0 = 1 \Rightarrow P_0(1 \mid 2) \quad t(x) = 9x - 7$
	

A5 Ausführliche Lösung (Berechnungen)

$$f(x)=\frac{1}{2}x^3-\frac{1}{2}x^2-\frac{5}{2}x+3 \qquad x_0=1 \qquad t(x)=f'(x_0)(x-x_0)+f(x_0)$$

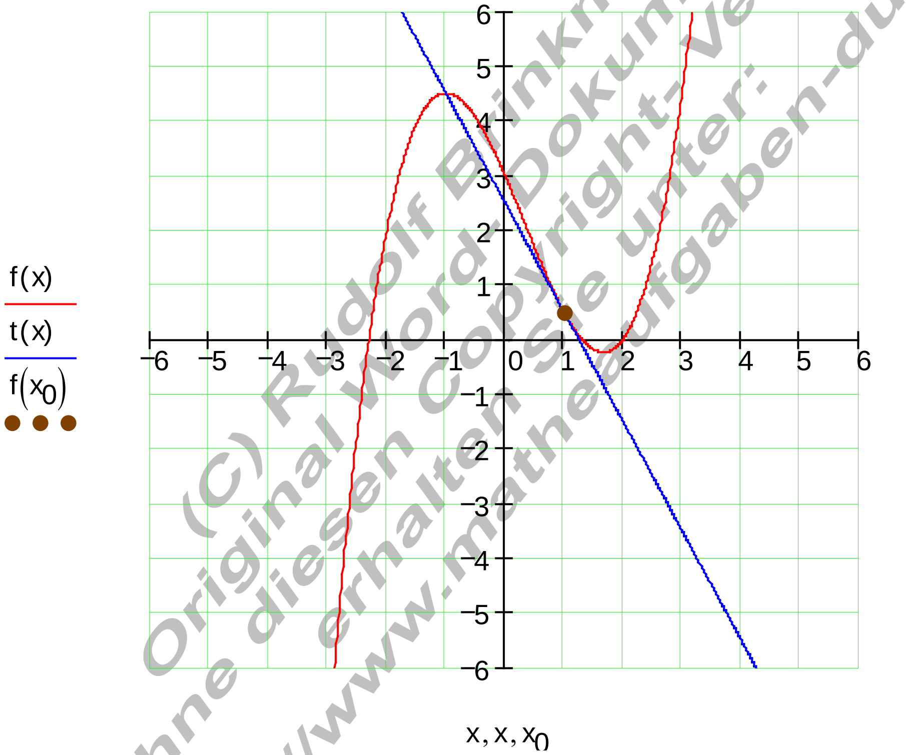
$$f'(x)=\frac{3}{2}x^2-x-\frac{5}{2}\Rightarrow f'(x_0)=f'(1)=\frac{3}{2}-1-\frac{5}{2}=-2$$

$$f(x_0)=f(1)=\frac{1}{2}-\frac{1}{2}-\frac{5}{2}+3=-\frac{5}{2}+\frac{6}{2}=\frac{1}{2}\Rightarrow \underline{\underline{P_0\left(1\,\middle|\,\frac{1}{2}\right)}}$$

$$t(x)=-2(x-1)+\frac{1}{2}=-2x+2+\frac{1}{2}=\underline{\underline{-2x+\frac{5}{2}}}$$

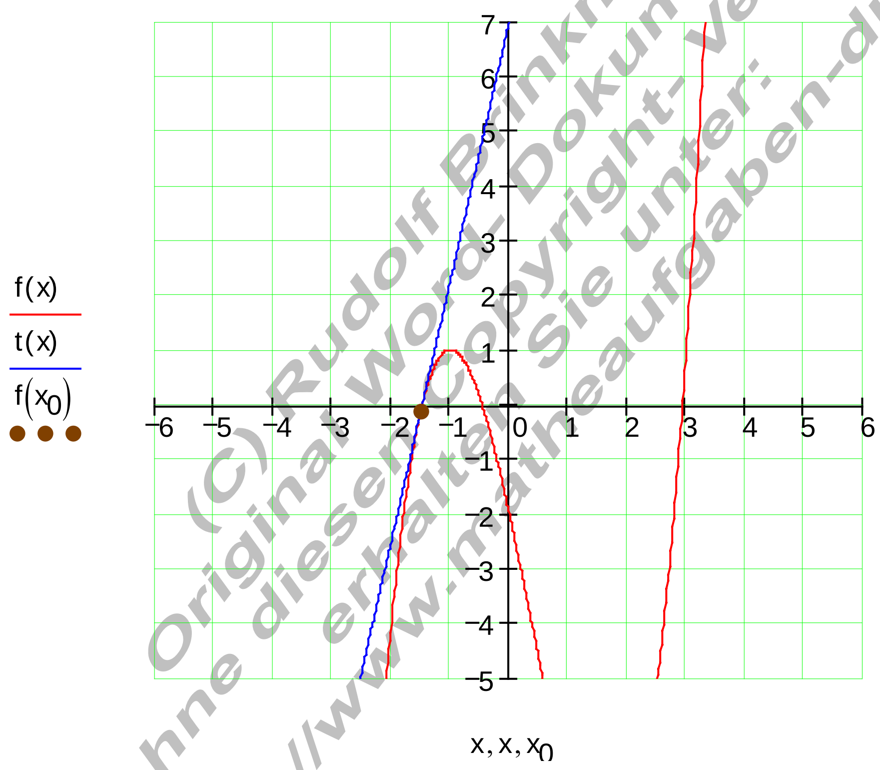
A5 Ausführliche Lösung (Die Graphen)

$$f(x)=\frac{1}{2}x^3-\frac{1}{2}x^2-\frac{5}{2}x+3 \qquad x_0=1\Rightarrow P_0\left(1\,\middle|\,\frac{1}{2}=0{,}5\right) \qquad t(x)=-2x+\frac{5}{2}$$

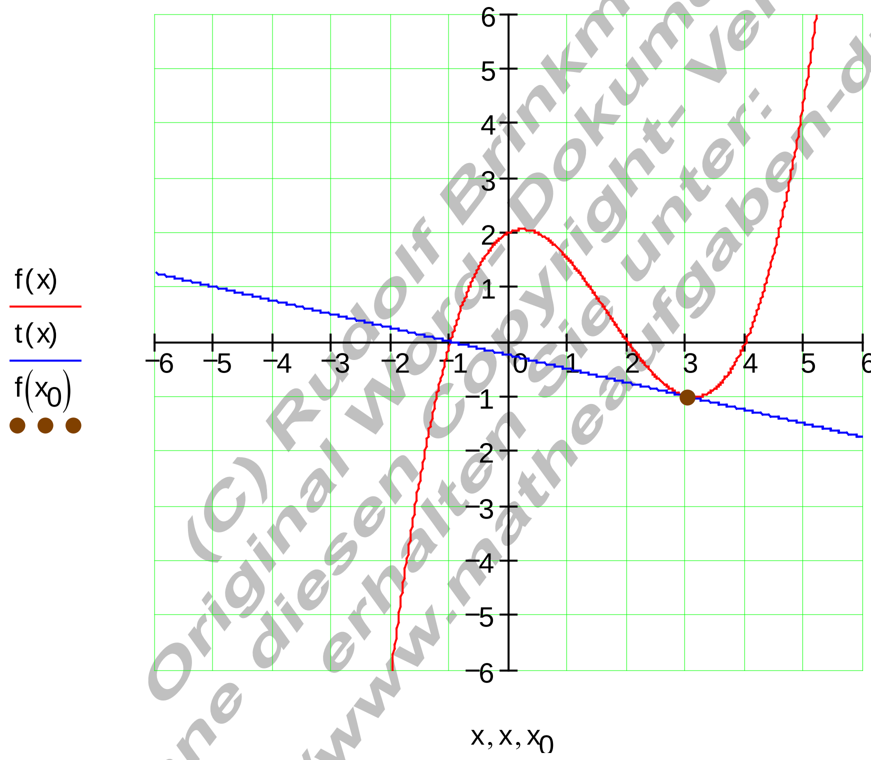


A6	Ausführliche Lösung (Berechnungen)
	$f(x)=x^3-x^2-5x-2 \quad x_0=-\frac{3}{2} \qquad t(x)=f'(x_0)(x-x_0)+f(x_0)$
	$f'(x)=3x^2-2x-5 \Rightarrow f'(x_0)=f'\left(-\frac{3}{2}\right)=3\cdot\frac{9}{4}-2\cdot\left(-\frac{3}{2}\right)-5=\frac{27}{4}+\frac{12}{4}-\frac{20}{4}=\frac{19}{4}$
	$f(x_0)=f\left(-\frac{3}{2}\right)=-\frac{27}{8}-\frac{9}{4}+\frac{15}{2}-2=-\frac{27}{8}-\frac{18}{8}+\frac{60}{8}-\frac{16}{8}=-\frac{1}{8} \Rightarrow \underline{\underline{P_0\left(-\frac{3}{2}\,\middle|\,-\frac{1}{8}\right)}}$
	$t(x)=\frac{19}{4}\left(x+\frac{3}{2}\right)-\frac{1}{8}=\frac{19}{4}x+\frac{57}{8}-\frac{1}{8}=\frac{19}{4}x+\frac{56}{8}=\underline{\underline{\frac{19}{4}x+7}}$

A6	Ausführliche Lösung (Die Graphen)
	$f(x)=x^3-x^2-5x-2 \quad x_0=-\frac{3}{2} \Rightarrow P_0\left(-\frac{3}{2}\,\middle|\,-\frac{1}{8}\right) \quad t(x)=\frac{19}{4}x+7$



A7	Ausführliche Lösung (Berechnungen)
	$f(x)=\frac{1}{4}x^3-\frac{5}{4}x^2+\frac{1}{2}x+2 \qquad x_0=3 \qquad t(x)=f'(x_0)(x-x_0)+f(x_0)$
	$f'(x)=\frac{3}{4}x^2-\frac{5}{2}x+\frac{1}{2} \Rightarrow f'(x_0)=f'(3)=\frac{27}{4}-\frac{15}{2}+\frac{1}{2}=\frac{27}{4}-\frac{30}{4}+\frac{2}{4}=-\frac{1}{4}$
	$f(x_0)=f(3)=\frac{27}{4}-\frac{45}{4}+\frac{3}{2}+2=\frac{27}{4}-\frac{45}{4}+\frac{6}{4}+\frac{8}{4}=-\frac{4}{4}-1 \Rightarrow \underline{\underline{P_0(3\mid -1)}}$
	$t(x)=-\frac{1}{4}(x-3)-1=-\frac{1}{4}x+\frac{3}{4}-1=-\frac{1}{4}x+\frac{3}{4}-\frac{4}{4}=\underline{\underline{-\frac{1}{4}x-\frac{1}{4}}}$

A7	Ausführliche Lösung (Die Graphen)
	$f(x)=\frac{1}{4}x^3-\frac{5}{4}x^2+\frac{1}{2}x+2 \qquad x_0=3 \Rightarrow P_0(3\mid -1) \qquad t(x)=-\frac{1}{4}x-\frac{1}{4}$
	

A8 Ausführliche Lösung (Berechnungen)

$$f(x) = x^3 - \frac{3}{2}x^2 - 6x + 2 \qquad x_0 = 1 \qquad t(x) = f'(x_0)(x - x_0) + f(x_0)$$

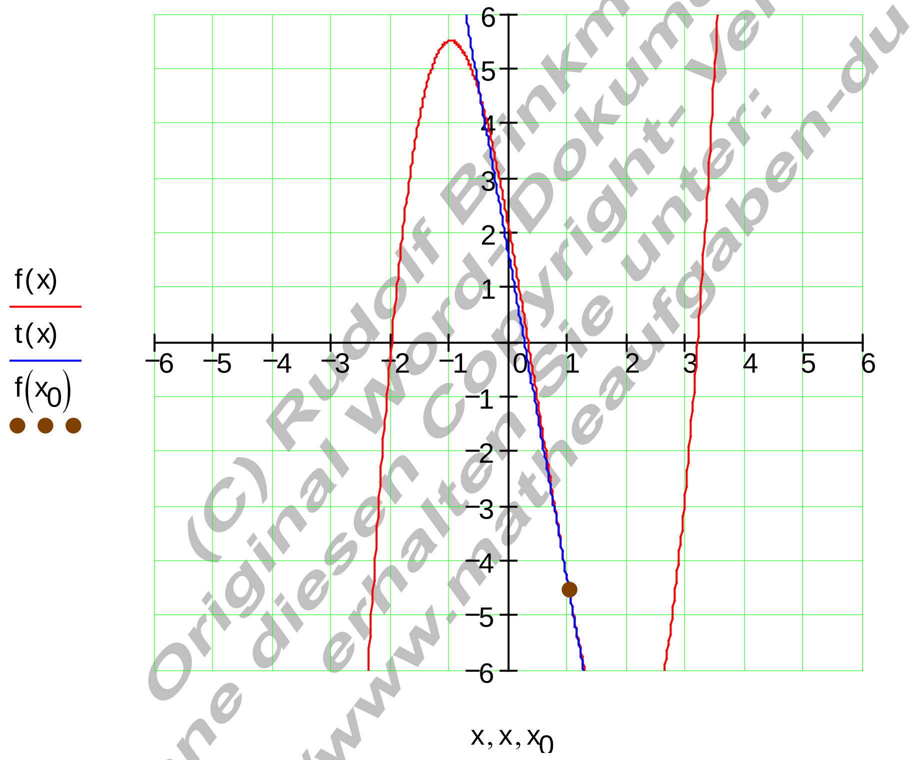
$$f'(x) = 3x^2 - 3x - 6 \Rightarrow f'(x_0) = f'(1) = 3 - 3 - 6 = -6$$

$$f(x_0) = f(1) = 1 - \frac{3}{2} - 6 + 2 = \frac{2}{2} - \frac{3}{2} - \frac{12}{2} + \frac{4}{2} = -\frac{9}{2} \Rightarrow \underline{\underline{P_0\left(1 \,\middle|\, -\frac{9}{2}\right)}}$$

$$t(x) = -6(x - 1) - \frac{9}{2} = -6x + 6 - \frac{9}{2} = -6x + \frac{12}{2} - \frac{9}{2} = \underline{\underline{-6x + \frac{3}{2}}}$$

A8 Ausführliche Lösung (Die Graphen)

$$f(x) = x^3 - \frac{3}{2}x^2 - 6x + 2 \qquad x_0 = 1 \Rightarrow P_0\left(1 \,\middle|\, -\frac{9}{2} = -4{,}5\right) \qquad t(x) = -6x + \frac{3}{2}$$



A9 Ausführliche Lösung (Berechnungen)

$f(x) = \frac{1}{2}x^3 - \frac{1}{2}x^2 - 4x + 4 \qquad x_0 = 0 \qquad t(x) = f'(x_0)(x - x_0) + f(x_0)$

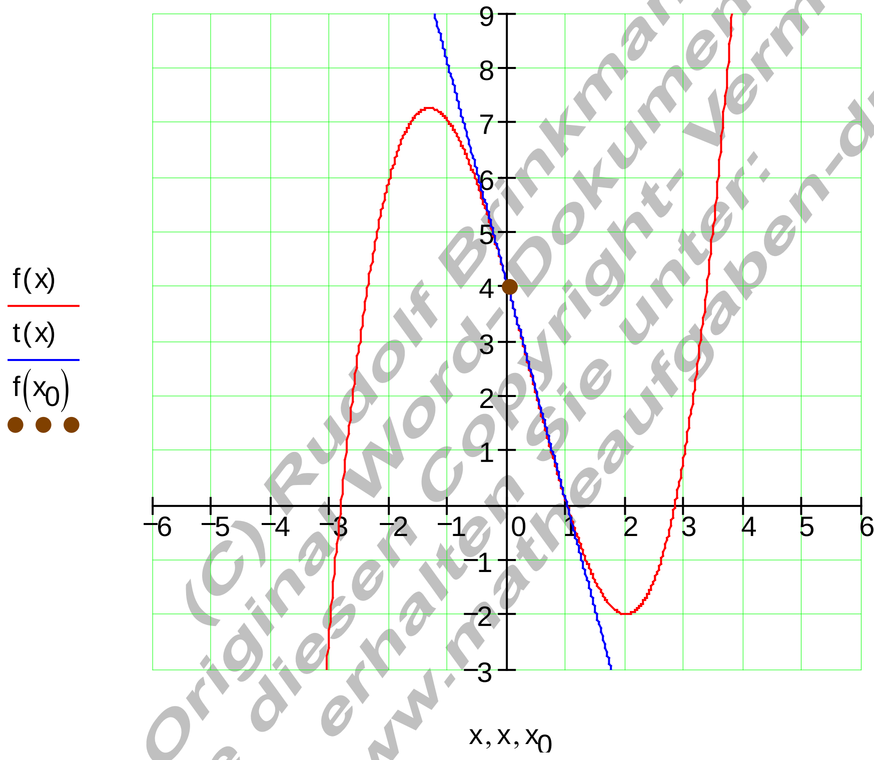
$f'(x) = \frac{3}{2}x^2 - x - 4 \Rightarrow f'(x_0) = f'(0) = -4$

$f(x_0) = f(0) = 4 \Rightarrow \underline{\underline{P_0(0\,|\,-4)}}$

$t(x) = -4(x - 0) + 4 = \underline{\underline{-4x + 4}}$

A9 Ausführliche Lösung (Die Graphen)

$f(x) = \frac{1}{2}x^3 - \frac{1}{2}x^2 - 4x + 4 \qquad x_0 = 0 \Rightarrow P_0(0\,|\,4) \qquad t(x) = -4x + 4$



A10	Ausführliche Lösung (Berechnungen)
	$f(x)=\frac{1}{3}x^3-\frac{1}{3}x^2-4x+4 \qquad x_0=1 \qquad t(x)=f'(x_0)(x-x_0)+f(x_0)$ $f'(x)=x^2-\frac{2}{3}x-4 \Rightarrow f'(x_0)=f'(1)=1-\frac{2}{3}-4=\frac{3}{3}-\frac{2}{3}-\frac{12}{3}=-\frac{11}{3}$ $f(x_0)=f(1)=\frac{1}{3}-\frac{1}{3}-4+4=\frac{1}{3}-\frac{1}{3}-\frac{12}{3}+\frac{12}{3}=0 \Rightarrow \underline{\underline{P_0(1\mid 0)}}$ $t(x)=-\frac{11}{3}(x-1)+0=\underline{\underline{-\frac{11}{3}x+\frac{11}{3}}}$

A10	Ausführliche Lösung (Die Graphen)
	$f(x)=\frac{1}{3}x^3-\frac{1}{3}x^2-4x+4 \qquad x_0=1 \Rightarrow P_0(1\mid 0) \qquad t(x)=-\frac{11}{3}x+\frac{11}{3}$