

Lösungen Training Differentialrechnung III

Ergebnisse:

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| E1 | Ergebnis: $f(x) = x^3 - 6x^2 + 9x \quad x_0 = 2 \Rightarrow P_0(2 2) \quad t(x) = -3x + 8$ |
| E2 | Ergebnis: $f(x) = \frac{1}{4}x^3 - \frac{9}{4}x^2 + \frac{15}{4}x + \frac{9}{4} \quad x_0 = 2 \Rightarrow P_0\left(2 \mid \frac{11}{4} = 2,75\right) \quad t(x) = -\frac{9}{4}x + \frac{29}{4}$ |
| E3 | Ergebnis: $f(x) = \frac{1}{2}x^3 - \frac{1}{2}x^2 - 4x + 4 \quad x_0 = -1 \Rightarrow P_0(-1 7) \quad t(x) = -\frac{3}{2}x + \frac{11}{2}$ |
| E4 | Ergebnis: $f(x) = x^3 + 3x^2 - 2 \quad x_0 = 1 \Rightarrow P_0(1 2) \quad t(x) = 9x - 7$ |
| E5 | Ergebnis: $f(x) = \frac{1}{2}x^3 - \frac{1}{2}x^2 - \frac{5}{2}x + 3 \quad x_0 = 1 \Rightarrow P_0\left(1 \mid \frac{1}{2} = 0,5\right) \quad t(x) = -2x + \frac{5}{2}$ |
| E6 | Ergebnis: $f(x) = x^3 - x^2 - 5x - 2 \quad x_0 = -\frac{3}{2} \Rightarrow P_0\left(-\frac{3}{2} \mid -\frac{1}{8}\right) \quad t(x) = \frac{19}{4}x + 7$ |
| E7 | Ergebnis: $f(x) = \frac{1}{4}x^3 - \frac{5}{4}x^2 + \frac{1}{2}x + 2 \quad x_0 = 3 \Rightarrow P_0(3 -1) \quad t(x) = -\frac{1}{4}x - \frac{1}{4}$ |
| E8 | Ergebnis: $f(x) = x^3 - \frac{3}{2}x^2 - 6x + 2 \quad x_0 = 1 \Rightarrow P_0\left(1 \mid -\frac{9}{2}\right) \quad t(x) = -6x + \frac{3}{2}$ |
| E9 | Ergebnis: $f(x) = \frac{1}{2}x^3 - \frac{1}{2}x^2 - 4x + 4 \quad x_0 = 0 \Rightarrow P_0(0 4) \quad t(x) = -4x + 4$ |
| E10 | Ergebnis: $f(x) = \frac{1}{3}x^3 - \frac{1}{3}x^2 - 4x + 4 \quad x_0 = 1 \Rightarrow P_0(1 0) \quad t(x) = -\frac{11}{3}x + \frac{11}{3}$ |

Ausführliche Lösungen:

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| A1 | Ausführliche Lösung (Berechnungen) |
| | $f(x) = x^3 - 6x^2 + 9x \quad x_0 = 2 \quad t(x) = f'(x_0)(x - x_0) + f(x_0)$ $f'(x) = 3x^2 - 12x + 9 \Rightarrow f'(x_0) = f'(2) = 12 - 24 + 9 = -3$ $f(x_0) = f(2) = 8 - 24 + 18 = 2 \Rightarrow \underline{\underline{P_0(2 2)}}$ $t(x) = -3(x - 2) + 2 = -3x + 6 + 2 = \underline{\underline{-3x + 8}}$ |

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| A1 | Ausführliche Lösung (Die Graphen) |
| | $f(x) = x^3 - 6x^2 + 9x \quad x_0 = 2 \Rightarrow P_0(2 2) \quad t(x) = -3x + 8$ |
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| A2 | Ausführliche Lösung (Berechnungen) |
| | $f(x) = \frac{1}{4}x^3 - \frac{9}{4}x^2 + \frac{15}{4}x + \frac{9}{4} \quad x_0 = 2 \quad t(x) = f'(x_0)(x - x_0) + f(x_0)$ $f'(x) = \frac{3}{4}x^2 - \frac{9}{2}x + \frac{15}{4} \Rightarrow f'(x_0) = f'(2) = \frac{3}{4} \cdot 4 - \frac{9}{2} \cdot 2 + \frac{15}{4} = \frac{12}{4} - \frac{36}{4} + \frac{15}{4} = -\frac{9}{4}$ $f(x_0) = f(2) = \frac{1}{4} \cdot 8 - \frac{9}{4} \cdot 4 + \frac{15}{4} \cdot 2 + \frac{9}{4} = \frac{8}{4} - \frac{36}{4} + \frac{30}{4} + \frac{9}{4} = \frac{11}{4} \Rightarrow P_0 \left(2 \mid \frac{11}{4} = 2,75 \right)$ $t(x) = -\frac{9}{4}(x - 2) + \frac{11}{4} = -\frac{9}{4}x + \frac{18}{4} + \frac{11}{4} = -\frac{9}{4}x + \frac{29}{4}$ |

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| A2 | Ausführliche Lösung (Die Graphen) |
| | $f(x) = \frac{1}{4}x^3 - \frac{9}{4}x^2 + \frac{15}{4}x + \frac{9}{4} \quad x_0 = 2 \Rightarrow P_0 \left(2 \mid \frac{11}{4} = 2,75 \right) \quad t(x) = -\frac{9}{4}x + \frac{29}{4}$ |
| | <p style="text-align: center;"> $f(x)$ $t(x)$ $f(x_0)$ ● ● ● </p> <p style="text-align: center;">x, x_0</p> |

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| A3 | Ausführliche Lösung (Berechnungen) |
| | $f(x) = \frac{1}{2}x^3 - \frac{1}{2}x^2 - 4x + 4 \quad x_0 = -1 \quad t(x) = f'(x_0)(x - x_0) + f(x_0)$ $f'(x) = \frac{3}{2}x^2 - x - 4 \Rightarrow f'(x_0) = f'(-1) = \frac{3}{2} + 1 - 4 = \frac{3}{2} + \frac{2}{2} - \frac{8}{2} = -\frac{3}{2}$ $f(x_0) = f(-1) = \frac{1}{2} \cdot (-1) - \frac{1}{2} \cdot 1 - 4 \cdot (-1) + 4 = -\frac{1}{2} - \frac{1}{2} + \frac{8}{2} + \frac{8}{2} = 7 \Rightarrow \underline{\underline{P_0(-1 7)}}$ $t(x) = -\frac{3}{2}(x+1) + 7 = -\frac{3}{2}x - \frac{3}{2} + \frac{14}{2} = -\frac{3}{2}x + \frac{11}{2}$ |

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| A3 | Ausführliche Lösung (Die Graphen) |
| | $f(x) = \frac{1}{2}x^3 - \frac{1}{2}x^2 - 4x + 4 \quad x_0 = -1 \Rightarrow P_0(-1 7) \quad t(x) = -\frac{3}{2}x + \frac{11}{2}$ |
| | <p style="text-align: center;"> $f(x)$ $t(x)$ $f(x_0)$ </p> |

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| A4 | Ausführliche Lösung (Berechnungen) |
| | $f(x) = x^3 + 3x^2 - 2 \quad x_0 = 1 \quad t(x) = f'(x_0)(x - x_0) + f(x_0)$ $f'(x) = 3x^2 + 6x \Rightarrow f'(x_0) = f'(1) = 3 + 6 = 9$ $f(x_0) = f(1) = 1^3 + 3 \cdot 1^2 - 2 = 1 + 3 - 2 = 2 \Rightarrow \underline{\underline{P_0(1 2)}}$ $t(x) = 9(x - 1) + 2 = 9x - 9 + 2 = \underline{\underline{9x - 7}}$ |

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| A4 | Ausführliche Lösung (Die Graphen) |
| | $f(x) = x^3 + 3x^2 - 2 \quad x_0 = 1 \Rightarrow P_0(1 2) \quad t(x) = 9x - 7$ |
| | <p> $f(x)$ $t(x)$ $f(x_0)$ </p> <p>• • •</p> <p>x, x, x_0</p> |

A5 Ausführliche Lösung (Berechnungen)

$$f(x) = \frac{1}{2}x^3 - \frac{1}{2}x^2 - \frac{5}{2}x + 3 \quad x_0 = 1 \quad t(x) = f'(x_0)(x - x_0) + f(x_0)$$

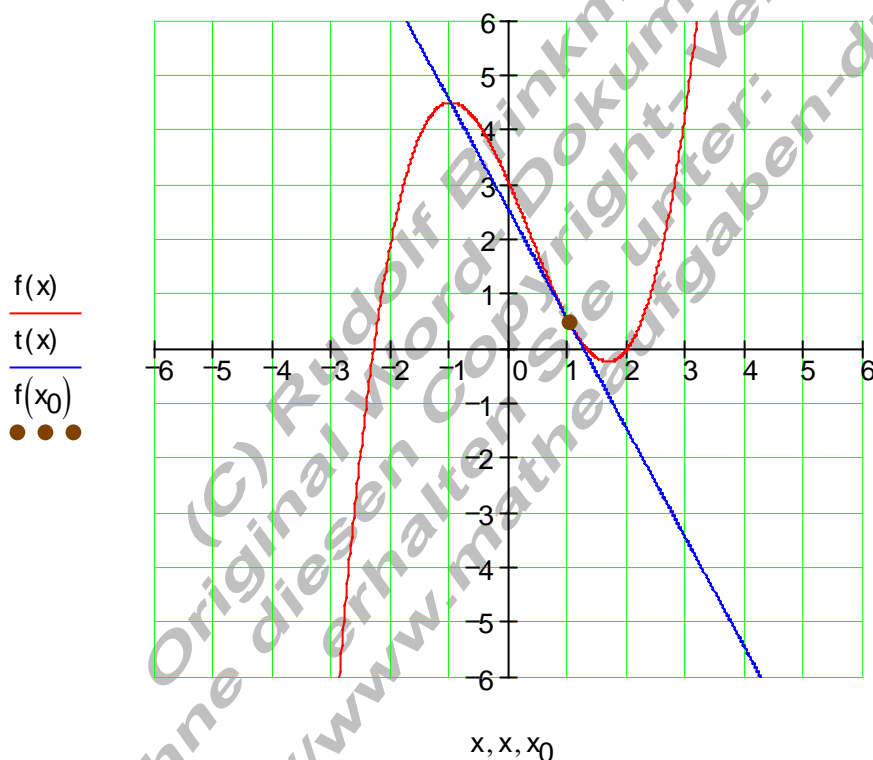
$$f'(x) = \frac{3}{2}x^2 - x - \frac{5}{2} \Rightarrow f'(x_0) = f'(1) = \frac{3}{2} - 1 - \frac{5}{2} = -2$$

$$f(x_0) = f(1) = \frac{1}{2} - \frac{1}{2} - \frac{5}{2} + 3 = -\frac{5}{2} + \frac{6}{2} = \frac{1}{2} \Rightarrow P_0 \left(1 \mid \frac{1}{2} \right)$$

$$t(x) = -2(x - 1) + \frac{1}{2} = -2x + 2 + \frac{1}{2} = -2x + \frac{5}{2}$$

A5 Ausführliche Lösung (Die Graphen)

$$f(x) = \frac{1}{2}x^3 - \frac{1}{2}x^2 - \frac{5}{2}x + 3 \quad x_0 = 1 \Rightarrow P_0 \left(1 \mid \frac{1}{2} = 0,5 \right) \quad t(x) = -2x + \frac{5}{2}$$



| A6 | Ausführliche Lösung (Berechnungen) |
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| | $f(x) = x^3 - x^2 - 5x - 2 \quad x_0 = -\frac{3}{2} \quad t(x) = f'(x_0)(x - x_0) + f(x_0)$ $f'(x) = 3x^2 - 2x - 5 \Rightarrow f'(x_0) = f'\left(-\frac{3}{2}\right) = 3 \cdot \frac{9}{4} - 2 \cdot \left(-\frac{3}{2}\right) - 5 = \frac{27}{4} + \frac{12}{4} - \frac{20}{4} = \frac{19}{4}$ $f(x_0) = f\left(-\frac{3}{2}\right) = -\frac{27}{8} - \frac{9}{4} + \frac{15}{2} - 2 = -\frac{27}{8} - \frac{18}{8} + \frac{60}{8} - \frac{16}{8} = -\frac{1}{8} \Rightarrow P_0\left(-\frac{3}{2} \mid -\frac{1}{8}\right)$ $t(x) = \frac{19}{4}\left(x + \frac{3}{2}\right) - \frac{1}{8} = \frac{19}{4}x + \frac{57}{8} - \frac{1}{8} = \frac{19}{4}x + \frac{56}{8} = \frac{19}{4}x + 7$ |

| A6 | Ausführliche Lösung (Die Graphen) |
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| | $f(x) = x^3 - x^2 - 5x - 2 \quad x_0 = -\frac{3}{2} \Rightarrow P_0\left(-\frac{3}{2} \mid -\frac{1}{8}\right) \quad t(x) = \frac{19}{4}x + 7$ |
| | <p> $f(x)$ $t(x)$ $f(x_0)$ </p> <p>• • •</p> <p>x, x, x_0</p> |

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| A7 | Ausführliche Lösung (Berechnungen) |
| $f(x) = \frac{1}{4}x^3 - \frac{5}{4}x^2 + \frac{1}{2}x + 2 \quad x_0 = 3 \quad t(x) = f'(x_0)(x - x_0) + f(x_0)$ | |
| $f'(x) = \frac{3}{4}x^2 - \frac{5}{2}x + \frac{1}{2} \Rightarrow f'(x_0) = f'(3) = \frac{27}{4} - \frac{15}{2} + \frac{1}{2} = \frac{27}{4} - \frac{30}{4} + \frac{2}{4} = -\frac{1}{4}$ | |
| $f(x_0) = f(3) = \frac{27}{4} - \frac{45}{4} + \frac{3}{2} + 2 = \frac{27}{4} - \frac{45}{4} + \frac{6}{4} + \frac{8}{4} = -\frac{4}{4} - 1 \Rightarrow \underline{\underline{P_0(3 -1)}}$ | |
| $t(x) = -\frac{1}{4}(x-3) - 1 = -\frac{1}{4}x + \frac{3}{4} - 1 = -\frac{1}{4}x + \frac{3}{4} - \frac{4}{4} = -\frac{1}{4}x - \frac{1}{4}$ | |

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| A7 | Ausführliche Lösung (Die Graphen) |
| $f(x) = \frac{1}{4}x^3 - \frac{5}{4}x^2 + \frac{1}{2}x + 2 \quad x_0 = 3 \Rightarrow P_0(3 -1) \quad t(x) = -\frac{1}{4}x - \frac{1}{4}$ | |
| <p>The graph displays a coordinate system with a green grid. The x-axis is labeled 'x, x, x₀' and ranges from -6 to 6. The y-axis is labeled 'f(x), t(x), f(x₀)' and ranges from -6 to 6. A red curve represents the function f(x), and a blue straight line represents the tangent line t(x) at the point P₀(3 -1). The point P₀ is marked with a brown dot at the coordinates (3, -1). The function f(x) has a local maximum at (0, 2) and a local minimum at (3, -1). The tangent line t(x) passes through (3, -1) and has a negative slope.</p> | |

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| A8 | Ausführliche Lösung (Berechnungen) $f(x) = x^3 - \frac{3}{2}x^2 - 6x + 2 \quad x_0 = 1 \quad t(x) = f'(x_0)(x - x_0) + f(x_0)$ $f'(x) = 3x^2 - 3x - 6 \Rightarrow f'(x_0) = f'(1) = 3 - 3 - 6 = -6$ $f(x_0) = f(1) = 1 - \frac{3}{2} - 6 + 2 = \frac{2}{2} - \frac{3}{2} - \frac{12}{2} + \frac{4}{2} = -\frac{9}{2} \Rightarrow P_0 \left(1 \mid -\frac{9}{2} \right)$ $t(x) = -6(x - 1) - \frac{9}{2} = -6x + 6 - \frac{9}{2} = -6x + \frac{12}{2} - \frac{9}{2} = -6x + \frac{3}{2}$ |
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| A8 | Ausführliche Lösung (Die Graphen) $f(x) = x^3 - \frac{3}{2}x^2 - 6x + 2 \quad x_0 = 1 \Rightarrow P_0 \left(1 \mid -\frac{9}{2} = -4,5 \right) \quad t(x) = -6x + \frac{3}{2}$ |
| | <p style="text-align: center;">x, x, x₀</p> |

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| A9 | Ausführliche Lösung (Berechnungen) $f(x) = \frac{1}{2}x^3 - \frac{1}{2}x^2 - 4x + 4 \quad x_0 = 0 \quad t(x) = f'(x_0)(x - x_0) + f(x_0)$ $f'(x) = \frac{3}{2}x^2 - x - 4 \Rightarrow f'(x_0) = f'(0) = -4$ $f(x_0) = f(0) = 4 \Rightarrow \underline{\underline{P_0(0 4)}}$ $t(x) = -4(x - 0) + 4 = \underline{\underline{-4x + 4}}$ |
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| A9 | Ausführliche Lösung (Die Graphen) $f(x) = \frac{1}{2}x^3 - \frac{1}{2}x^2 - 4x + 4 \quad x_0 = 0 \Rightarrow P_0(0 4) \quad t(x) = -4x + 4$ |
| | <p style="text-align: center;">x, x_0</p> |

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| A10 | Ausführliche Lösung (Berechnungen) |
| | $f(x) = \frac{1}{3}x^3 - \frac{1}{3}x^2 - 4x + 4 \quad x_0 = 1 \quad t(x) = f'(x_0)(x - x_0) + f(x_0)$ $f'(x) = x^2 - \frac{2}{3}x - 4 \Rightarrow f'(x_0) = f'(1) = 1 - \frac{2}{3} - 4 = \frac{3}{3} - \frac{2}{3} - \frac{12}{3} = -\frac{11}{3}$ $f(x_0) = f(1) = \frac{1}{3} - \frac{1}{3} - 4 + 4 = \frac{1}{3} - \frac{1}{3} - \frac{12}{3} + \frac{12}{3} = 0 \Rightarrow \underline{\underline{P_0(1 0)}}$ $t(x) = -\frac{11}{3}(x-1) + 0 = \underline{\underline{-\frac{11}{3}x + \frac{11}{3}}}$ |

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| A10 | Ausführliche Lösung (Die Graphen) |
| | $f(x) = \frac{1}{3}x^3 - \frac{1}{3}x^2 - 4x + 4 \quad x_0 = 1 \Rightarrow P_0(1 0) \quad t(x) = -\frac{11}{3}x + \frac{11}{3}$ |
| | <p> $f(x)$ $t(x)$ $f(x_0)$ $\bullet \bullet \bullet$ </p> <p style="text-align: center;">x, x_0</p> |