Lösungen Training ganzrationale Funktionen VIII

Ausführliche Lösungen:

A1 Ausführliche Lösung

grad 3, punktsymmetrisch durch

$P_1(2|3)\ \ P_2(-3|-2)$

Wegen der Punktsymmetrie hat die Funktionsgleichung nur ungerade Exponenten.

Ansatz: $f(x) = a_3x^3 + a_1x$

$P_1(2|3) \quad \Rightarrow f(2) = 8a_3 + 2a_1 = 3$

$P_2(-3|-2) \Rightarrow f(-3) = -27a_3 - 3a_1 = -2$

a_1	a_3		
2	8	3	$\mid\cdot 3$
−3	−27	−2	$\mid\cdot 2$
6	24	9	
−6	−54	−4	II + I
6	24	9	
0	−30	5	

$-30a_3 = 5$

$\Leftrightarrow a_3 = -\frac{1}{6}$

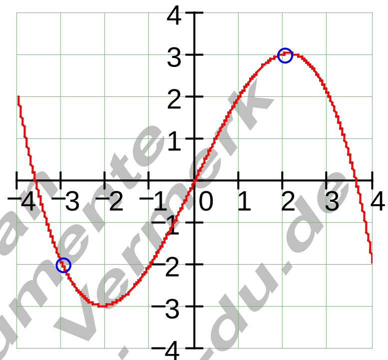
$6a_1 + 24a_3 = 9$

$\Leftrightarrow 6a_1 - 4 = 9$

$\Leftrightarrow a_1 = \frac{13}{6}$

$$\underline{\underline{f(x) = -\frac{1}{6}x^3 + \frac{13}{6}x}}$$

$f(x) := \frac{-1}{6}\cdot x^3 + \frac{13}{6}\cdot x$



A2 Ausführliche Lösung

grad 3, Nullstellen $x_1 = -3;\ x_2 = -1;\ x_3 = 2$

Punkt $P(-2|2)$

Ansatz über Linearfaktoren

$f(x) = a_3(x - x_1)(x - x_2)(x - x_3)$

$f(x) = a_3(x + 3)(x + 1)(x - 2)$

Punktprobe:

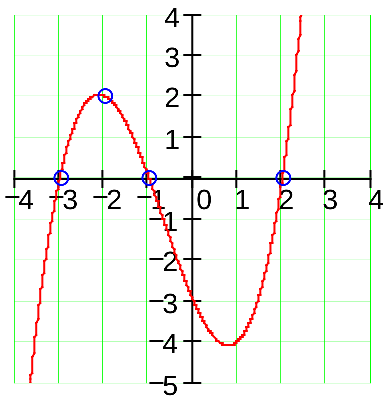
$f(-2) = 2 \Leftrightarrow a_3(-2 + 3)(-2 + 1)(-2 - 2) = 2$

$\Leftrightarrow 4a_3 = 2 \Leftrightarrow a_3 = \frac{1}{2}$

$f(x) = \frac{1}{2}(x + 3)(x + 1)(x - 2)$

$$\underline{\underline{= \frac{1}{2}x^3 + x^2 - \frac{5}{2}x - 3}}$$

$f(x) := \frac{1}{2}\cdot x^3 + x^2 - \frac{5}{2}\cdot x - 3$



A3 Ausführliche Lösung

grad 3, Nullstellen $x_{1/2} = 0\,;\; x_3 = 2\,;$

Punkt $P(1|5)$

Ansatz über Linearfaktoren

$f(x) = a_3 x^2 (x - x_3)$

$f(x) = a_3 x^2 (x - 2)$

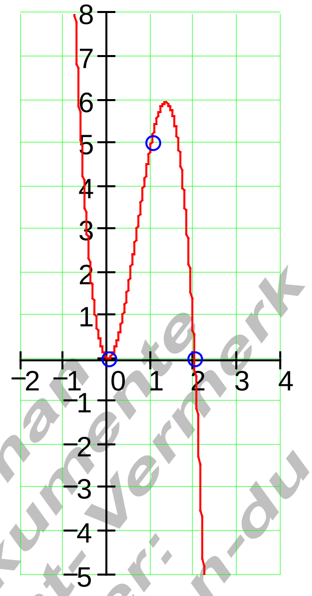
Punktprobe :

$f(1) = 5 \Leftrightarrow a_3 \cdot 1^2 (1-2) = 5$

$\Leftrightarrow -a_3 = 5 \Leftrightarrow a_3 = -5$

$f(x) = -5x^2 (x-2) = \underline{\underline{-5x^3 + 10x^2}}$

$f(x) := -5 \cdot x^3 + 10 \cdot x^2$



A4 Ausführliche Lösung

grad 3, Nullstellen $x_{1/2} = -2\,;\; x_3 = 1$

Punkt $P(2|4)$

Ansatz über Linearfaktoren

$f(x) = a_3 (x - x_{1/2})^2 (x - x_3)$

$f(x) = a_3 (x + 2)^2 (x - 1)$

Punktprobe :

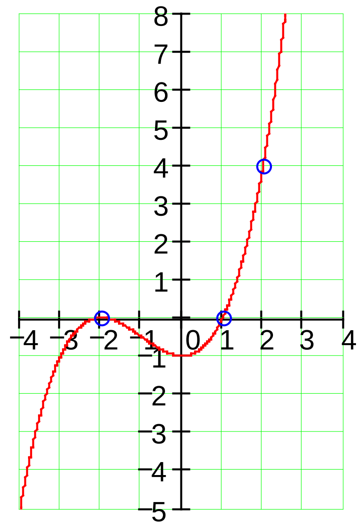
$f(2) = 4 \Leftrightarrow a_3 \cdot (2+2)^2 (2-1) = 4$

$\Leftrightarrow 16a_3 = 4 \Leftrightarrow a_3 = \frac{1}{4}$

$f(x) = \frac{1}{4}(x+2)^2 (x-1)$

$= \underline{\underline{\frac{1}{4}x^3 + \frac{3}{4}x^2 - 1}}$

$f(x) := \frac{1}{4} \cdot x^3 + \frac{3}{4} \cdot x^2 - 1$



A5 Ausführliche Lösung

grad 3, Nullstellen $x_{1/2/3} = 3$

Punkt $P(-1|\,8)$

Ansatz über Linearfaktoren

$f(x) = a_3 (x - x_{1/2/3})^3$

$f(x) = a_3 (x-3)^2$

Punktprobe :

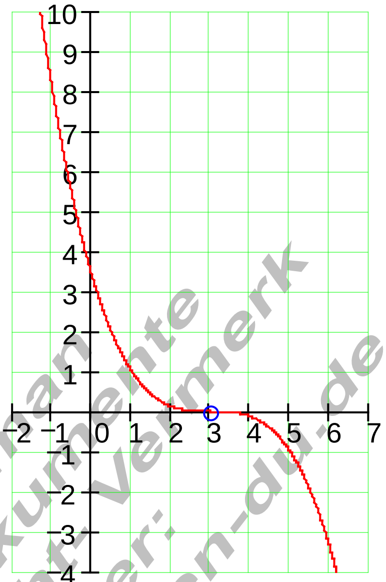
$f(-1) = 8 \Leftrightarrow a_3 \cdot (-1-3)^3 = 8$

$\Leftrightarrow -64a_3 = 8 \Leftrightarrow a_3 = -\frac{1}{8}$

$f(x) = -\frac{1}{8}(x-3)^3$

$= -\frac{1}{8}x^3 + \frac{9}{8}x^2 - \frac{27}{8}x + \frac{27}{8}$

$f(x) := \frac{-1}{8} \cdot x^3 + \frac{9}{8} \cdot x^2 - \frac{27}{8} \cdot x + \frac{27}{8}$



A6 Ausführliche Lösung

grad 4, achsensymmetrisch

$P_1(1|2);P_2(2|-1);P_3(-3|-2)$

Wegen der Achsensymmetrie hat die Funktionsgleichung nur gerade Exponenten.

Ansatz: $f(x)=a_4x^4+a_2x^2+a_0$

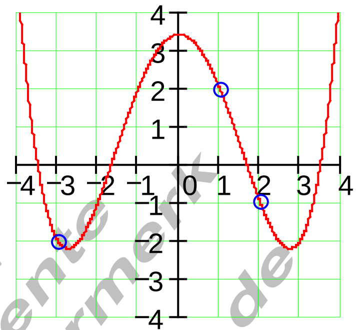
$P_1(1|2)\quad\Rightarrow f(1)=1a_4+1a_2+1a_0=2$

$P_2(2|-1)\Rightarrow f(2)=16a_4+4a_2+1a_0=-1$

$P_2(-3|-2)\Rightarrow f(-3)=81a_4+9a_2+1a_0=-2$

a_0	a_2	a_4		
1	1	1	2	
1	4	16	−1	II − I
1	9	81	−2	III − I
1	1	1	2	
0	3	15	−3	$\mid :3$
0	8	80	−4	$\mid :8$
1	1	1	2	
0	1	5	−1	
0	1	10	−1/2	III − II
1	1	1	2	
0	1	5	−1	
0	0	5	1/2	

$$f(x):=\frac{1}{10}\cdot x^4-\frac{3}{2}\cdot x^2+\frac{17}{5}$$



$5a_4=\frac{1}{2}\mid :5\Leftrightarrow a_4=\frac{1}{10}$

$a_2+5a_4=-1$

$\Leftrightarrow a_2+\frac{5}{10}=-\frac{10}{10}\mid -\frac{5}{10}$

$\Leftrightarrow a_2=-\frac{15}{10}=-\frac{3}{2}$

$a_0+a_2+a_4=2$

$\Leftrightarrow a_0-\frac{3}{2}+\frac{1}{10}=\frac{20}{10}\mid +\frac{15}{10}-\frac{1}{10}$

$\Leftrightarrow a_0=\frac{34}{10}=\frac{17}{5}$

$$\underline{\underline{f(x)=\frac{1}{10}x^4-\frac{3}{2}x^2+\frac{17}{5}}}$$

A7 Ausführliche Lösung

grad 4, Nullstellen $x_{1/2/3} = -2\,;\; x_4 = 2$

Punkt $P(1|3)$

Ansatz über Linearfaktoren

$f(x) = a_4\left(x - x_{1/2/3}\right)^3 \left(x - x_4\right)$

$f(x) = a_4\left(x + 2\right)^3 \left(x - 2\right)$

Punktprobe :

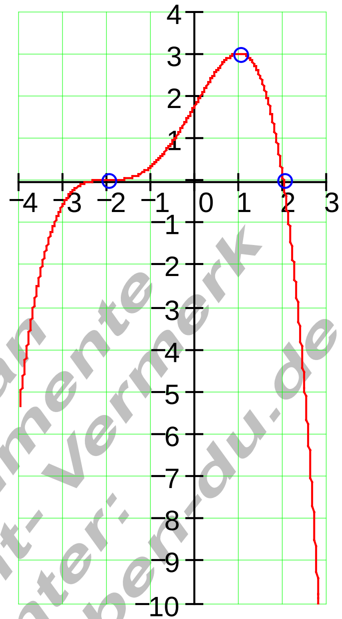
$f(1) = 3 \Leftrightarrow a_4\left(1+2\right)^3\left(1-2\right) = 3$

$\Leftrightarrow -27a_4 = 3 \Leftrightarrow a_4 = -\frac{1}{9}$

$f(x) = -\frac{1}{9}\left(x+2\right)^3\left(x-2\right)$

$= -\frac{1}{9}x^4 - \frac{4}{9}x^3 + \frac{16}{9}x + \frac{16}{9}$

$f(x) := \frac{-1}{9}\cdot x^4 - \frac{4}{9}\cdot x^3 + \frac{16}{9}\cdot x + \frac{16}{9}$



A8	Ausführliche Lösung

grad 4, durch den Ursprung $(a_0 = 0)$

$P_1(1|1); P_2(-1|-3); P_3(3|-1); P_4(-3|-1)$

Ansatz: $f(x) = a_4x^4 + a_3x^3 + a_2x^2 + a_1x$

$P_1(1|1) \quad : f(1) = 1a_4 + 1a_3 + 1a_2 + 1a_1 = 1$

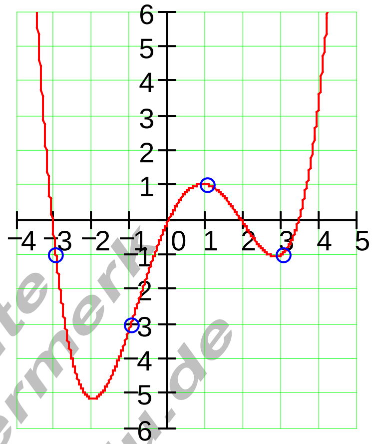
$P_1(-1|3) \quad : f(-1) = 1a_4 - 1a_3 + 1a_2 - 1a_1 = 3$

$P_1(3|-1) \quad : f(3) = 81a_4 + 27a_3 + 9a_2 + 3a_1 = -1$

$P_1(-3|-1) : f(-3) = 81a_4 - 27a_3 + 9a_2 - 3a_1 = -1$

a_1	a_2	a_3	a_4		
1	1	1	1	1	
–1	1	–1	1	–3	II + I
3	9	27	81	–1	III – 3·I
–3	9	–27	81	–1	IV + 3·I
1	1	1	1	1	
0	2	0	2	–2	
0	6	24	78	–4	III – 3·II
0	12	–24	84	2	IV – 6·II
1	1	1	1	1	
0	2	0	2	–2	
0	0	24	72	2	
0	0	–24	72	14	IV + III
1	1	1	1	1	
0	2	0	2	–2	
0	0	24	72	2	
0	0	0	144	16	

$f(x) := \frac{1}{9}\cdot x^4 - \frac{1}{4}\cdot x^3 - \frac{10}{9}\cdot x^2 + \frac{9}{4}\cdot x$



$144a_4 = 16 \Leftrightarrow a_4 = \frac{1}{9}$

$24a_3 + 72a_4 = 2$

$\Leftrightarrow 24a_3 + \frac{72}{9} = \frac{18}{9} \Leftrightarrow a_3 = -\frac{1}{4}$

$2a_2 + 2a_4 = -2$

$\Leftrightarrow 2a_2 + \frac{2}{9} = -\frac{18}{9} \Leftrightarrow a_2 = -\frac{10}{9}$

$a_1 + a_2 + a_3 + a_4 = 1$

$\Leftrightarrow a_1 - \frac{10}{9} - \frac{1}{4} + \frac{1}{9} = 1 \Leftrightarrow a_1 = \frac{9}{4}$

$$f(x) = \frac{1}{9}x^4 - \frac{1}{4}x^3 - \frac{10}{9}x^2 + \frac{9}{4}x$$

A9 Ausführliche Lösung

grad 4, Nullstellen

$x_1 = -3\,;\ x_2 = -1\,;\ x_3 = 2\,;\ x_4 = 5$

Punkt $P(1|\,2)$

Ansatz über Linearfaktoren

$f(x) = a_4\,(x - x_1)(x - x_2)(x - x_3)(x - x_4)$

$f(x) = a_4\,(x + 3)(x + 1)(x - 2)(x - 5)$

Punktprobe:

$f(1) = 2$

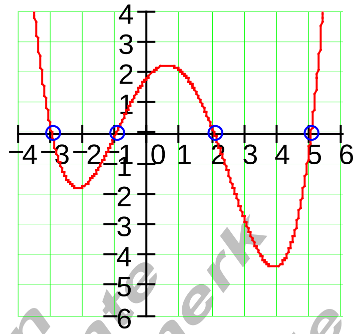
$\Leftrightarrow a_4\,(1+3)(1+1)(1-2)(1-5) = 2$

$\Leftrightarrow 32a_4 = 2 \Leftrightarrow a_4 = \frac{1}{16}$

$f(x) = \frac{1}{16}(x+3)(x+1)(x-2)(x-5)$

$= \frac{1}{16}x^4 - \frac{3}{16}x^3 - \frac{15}{16}x^2 + \frac{19}{16}x + \frac{15}{8}$

$f(x) = \frac{1}{16}x^4 - \frac{3}{16}x^3 - \frac{15}{16}x^2 + \frac{19}{16}x + \frac{15}{8}$



A10 Ausführliche Lösung

grad 4, Nullstellen

$x_{1/2} = -3\,;\ x_{3/4} = 2\,;\ \ P(1|\,4)$

Ansatz über Linearfaktoren

$f(x) = a_4\,(x - x_{1/2})^2\,(x - x_{3/4})^2$

$f(x) = a_4\,(x + 3)^2\,(x - 2)^2$

Punktprobe:

$f(1) = 4 \Leftrightarrow a_4\,(1+3)^2\,(1+2)^2 = 4$

$\Leftrightarrow 16a_4 = 4 \Leftrightarrow a_4 = \frac{1}{4}$

$f(x) = \frac{1}{4}(x+3)^2\,(x-2)^2$

$= \frac{1}{4}x^4 + \frac{1}{2}x^3 - \frac{11}{4}x^2 - 3x + 9$

$f(x) := \frac{1}{4}\cdot x^4 + \frac{1}{2}\cdot x^3 - \frac{11}{4}\cdot x^2 - 3\cdot x + 9$