Lösungen Training ganzrationale Funktionen IV

Ergebnisse:

1. Ergebnis

$f(x) = 0 \Leftrightarrow \frac{1}{2}x^4 - \frac{13}{2}x^2 + 18 = 0 \quad x^2 = z$

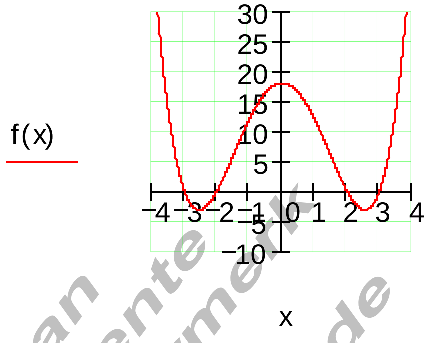
$\frac{1}{2}z^2 - \frac{13}{2}x + 18 = 0 \Leftrightarrow z^2 - 13z + 36 = 0$

$z_1 = 9\,;\, z_2 = 4 \Leftrightarrow x_{1/2} = \pm 3\,;\, x_{3/4} = \pm 2$

$P_{x1}(3\,|\,0)\,;\,P_{x2}(-3\,|\,0)$

$P_{x3}(2\,|\,0)\,;\,P_{x4}(-2\,|\,0)$

$f(x) = \frac{1}{2}(x-3)(x+3)(x-2)(x+2)$



2. Ergebnis

$f(x) = 0 \Leftrightarrow x^3 - 4x^2 + x + 6 = 0$

Nullstelle raten $x_1 = 2$

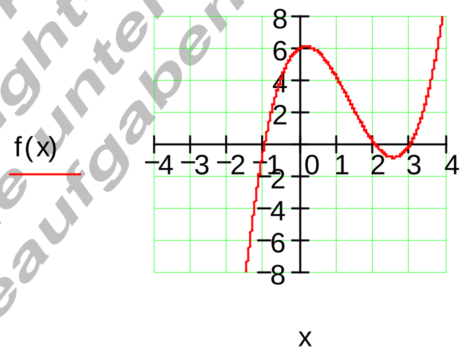
$\left(x^3 - 4x^2 + x + 6\right) : (x-2) = x^2 - 2x - 3$

$p = -2\,;\, q = -3 \Rightarrow D = 4$

$x_{2/3} = -\frac{p}{2} \pm \sqrt{D} \quad \Big| \quad x_2 = 1 + 2 = 3;\quad x_3 = 1 - 2 = -1$

$P_{x1}(2\,|\,0)\,;\,P_{x2}(3\,|\,0)\,;\,P_{x3}(-1\,|\,0)$

$f(x) = (x-2)(x-3)(x+1)$



3. Ergebnis

$f(x) = 0 \Leftrightarrow \frac{1}{2}x^2 - \frac{1}{2}x - 1 = 0$

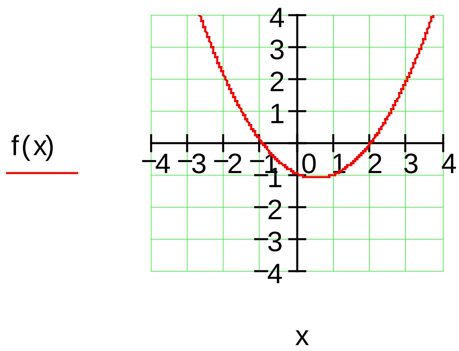
$x^2 - x - 2 = 0$

$p = -1\,;\, q = -2 \Rightarrow D = \frac{9}{4}$

$x_{1/2} = -\frac{p}{2} \pm \sqrt{D} \quad \Big| \quad x_1 = \frac{1}{2} + \frac{3}{2} = 2;\quad x_2 = \frac{1}{2} - \frac{3}{2} = -1$

$P_{x1}(2\,|\,0)\,;\,P_{x2}(-1\,|\,0)$

$f(x) = \frac{1}{2}(x-2)(x+1)$



4. Ergebnis

$$f(x)=0 \Leftrightarrow f(x)=-\frac{3}{2}x^4+\frac{75}{2}x^2-216=0$$

$$x^2=z \Rightarrow -\frac{3}{2}z^2+\frac{75}{2}z-216=0$$

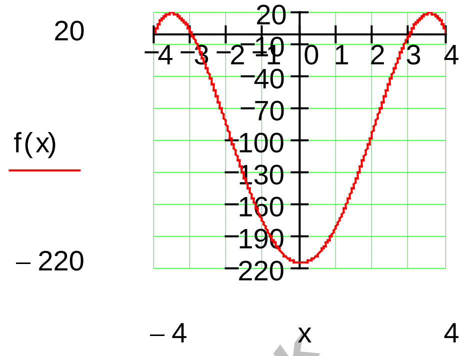
$$\Leftrightarrow z^2-25z+144=0$$

$$z_1=16\,;\,z_2=9 \Leftrightarrow x_{1/2}=\pm 4\,;\,x_{3/4}=\pm 3$$

$$P_{x1}(4\,|\,0)\,;\,P_{x2}(-4\,|\,0)$$

$$P_{x3}(3\,|\,0)\,;\,P_{x4}(-3\,|\,0)$$

$$f(x)=-\frac{3}{2}(x-4)(x+4)(x-3)(x+3)$$



5. Ergebnis

$$f(x)=0 \Leftrightarrow -\frac{1}{4}x^3-\frac{1}{2}x^2+\frac{11}{4}x+3=0$$

Nullstelle raten $x_1=3$

$$\left(-\frac{1}{4}x^3-\frac{1}{2}x^2+\frac{11}{4}x+3\right):(x-3)$$

$$=-\frac{1}{4}x^2-\frac{5}{4}x-1$$

$$x^2+5x+4=0$$

$$p=5\,;\,q=4 \Rightarrow D=\frac{9}{4}$$

$$x_{2/3}=-\frac{p}{2}\pm\sqrt{D} \quad \left|\; \begin{array}{l} x_2=-\frac{5}{2}+\frac{3}{2}=-1 \\ x_3=-\frac{5}{2}-\frac{3}{2}=-4 \end{array}\right.$$

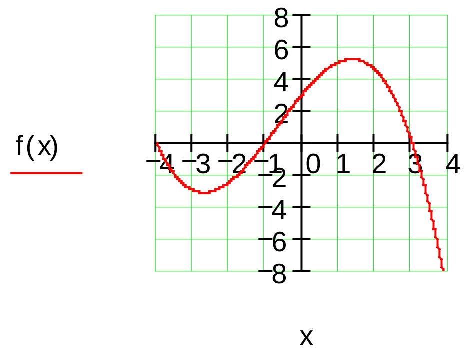
$$P_{x1}(3\,|\,0)\,;\,P_{x2}(-1\,|\,0)\,;\,P_{x3}(-4\,|\,0)$$

$$f(x)=-\frac{1}{4}(x-3)(x+1)(x+4)$$

Lösung mit dem Horner- Schema

	$-\frac{1}{4}$	$-\frac{1}{2}$	$\frac{11}{4}$	3	
$x=3$	↓	$-\frac{3}{4}$	$-\frac{15}{4}$	-3	
	$-\frac{1}{4}$	$-\frac{5}{4}$	-1	0	$=f(3)$

$-\frac{1}{4}x^2-\frac{5}{4}x-1=0$ Restpolynom



6. Ergebnis

$$f(x)=0 \Leftrightarrow -\frac{3}{4}x^3-\frac{3}{2}x^2+\frac{9}{4}x=0$$

x ausklammern:

$$x\left(-\frac{3}{4}x^2-\frac{3}{2}x+\frac{9}{4}\right)=0 \Rightarrow x_1=0$$

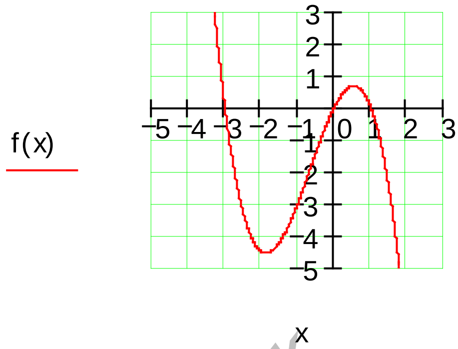
$$-\frac{3}{4}x^2-\frac{3}{2}x+\frac{9}{4}=0 \Leftrightarrow x^2+2x-3=0$$

$$p=2\,;\,q=-3 \Rightarrow D=4$$

$$x_{2/3}=-\frac{p}{2}\pm\sqrt{D} \quad \left|\; \begin{array}{l} x_2=-1+2=1 \\ x_3=-1-2=-3 \end{array}\right.$$

$$P_{x1}(0\,|\,0)\,;P_{x2}(1\,|\,0)\,;P_{x3}(-3\,|\,0)$$

$$f(x)=-\frac{3}{4}x(x-1)(x+3)$$



7. Ergebnis

$$f(x)=0 \Leftrightarrow 3x^4-3=0$$

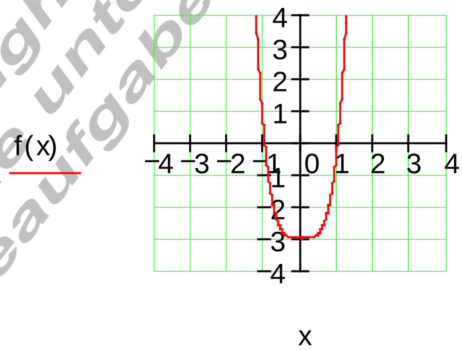
$$x^2=z$$

$$3z^2-3=0 \Leftrightarrow z^2-1=0$$

$$z_1=1;\,z_2=-1 \Leftrightarrow x_{1/2}=\pm 1$$

$$P_{x1}(1|0)\,;P_{x2}(-1|0)$$

Darstellung durch Linearfaktoren nicht möglich



8. Ergebnis

$f(x)=0 \Leftrightarrow \frac{1}{3}x^3+\frac{4}{3}x^2+\frac{1}{3}x-2=0$

Nullstelle raten $x_1=1$

$\left(\frac{1}{3}x^3+\frac{4}{3}x^2+\frac{1}{3}x-2\right):(x-1)$

$=\frac{1}{3}x^2+\frac{5}{3}x+2$

$x^2+5x+6=0$

$p=5\,;\,q=6 \Rightarrow D=\frac{1}{4}$

$x_{2/3}=-\frac{p}{2}\pm\sqrt{D}\quad\left|\begin{array}{l} x_2=-\frac{5}{2}+\frac{1}{2}=-2 \\ x_3=-\frac{5}{2}-\frac{1}{2}=-3\end{array}\right.$

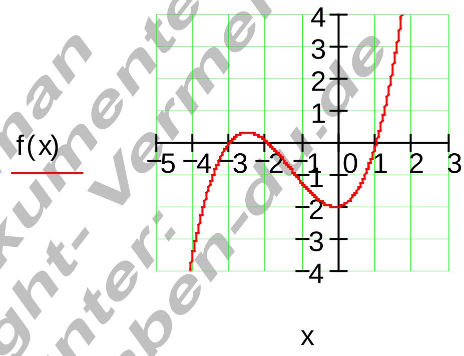
$P_{x1}(1\,|\,0)\,;P_{x2}(-2\,|\,0)\,;P_{x3}(-3\,|\,0)$

$f(x)=\frac{1}{3}(x-1)(x+2)(x+3)$

Lösung mit dem Horner- Schema

		$\frac{1}{3}$	$\frac{4}{3}$	$\frac{1}{3}$	-2
$x=1$	$\downarrow$	$\frac{1}{3}$	$\frac{5}{3}$	2	
	$\frac{1}{3}$	$\frac{5}{3}$	2	0	$=f(1)$

$\frac{1}{3}x^2+\frac{5}{3}x+2=0$ Restpolynom



9. Ergebnis

$f(x)=0 \Leftrightarrow 2x^3-4x^2-16x=0$

x ausklammern:

$x\left(2x^2-4x-16\right)=0 \Rightarrow x_1=0$

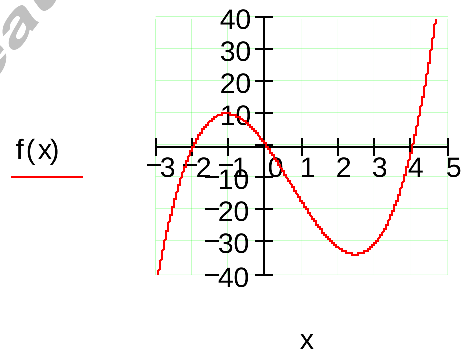
$2x^2-4x-16=0 \Leftrightarrow x^2-2x-8=0$

$p=-2\,;\,q=-8 \Rightarrow D=9$

$x_{2/3}=-\frac{p}{2}\pm\sqrt{D}\quad\left|\begin{array}{l} x_2=1+3=4 \\ x_3=1-3=-2\end{array}\right.$

$P_{x1}(0\,|\,0)\,;P_{x2}(4\,|\,0)\,;P_{x3}(-2\,|\,0)$

$f(x)=2x(x-4)(x+2)$



10. Ergebnis

$f(x) = 0 \Leftrightarrow 2x^3 + 4x^2 - 26x + 20 = 0$

Nullstelle raten $x_1 = 1$

$(2x^3 + 4x^2 - 26x + 20) : (x - 1)$

$= 2x^2 + 6x - 20$

$x^2 + 3x - 10 = 0$

$p = 3\,;\, q = -10 \Rightarrow D = \frac{49}{4}$

$x_{2/3} = -\frac{p}{2} \pm \sqrt{D}$ | $x_2 = -\frac{3}{2} + \frac{7}{2} = 2$; $x_3 = -\frac{3}{2} - \frac{7}{2} = -5$

$P_{x1}(1\,|\,0)\,;\, P_{x2}(2\,|\,0)\,;\, P_{x3}(-5\,|\,0)$

$f(x) = 2(x-1)(x-2)(x+5)$

Lösung mit dem Horner- Schema

		2	4	−26	20
x = 1	↓		2	6	−20
		2	6	−20	0 = f(1)

$2x^2 + 6x - 20 = 0$ Restpolynom