Trainingsaufgaben zur Abiturvorbereitung

9. Integration durch Substitution

Aufgaben

Lösen, bzw. berechnen Sie folgende Integrale.			
1.	$\int \frac{3}{4x+1}\,dx$	2.	$\int_0^2 \frac{4}{4-x}\,dx$
3.	$\int \frac{2}{(1-x)^2}\,dx$	4.	$\int \frac{6}{(2x-1)^3}\,dx$
5.	$\int_{-2}^{2} \frac{10}{(x-4)^5}\,dx$	6.	$\int_{-2}^{2} e^{1-x}\,dx$
7.	$\int_0^4 e^{\frac{1}{2}x}\,dx$	8.	$\int_1^2 e^{4-2x}\,dx$
9.	$\int_1^2 \frac{4}{e^{2x-4}}\,dx$	10.	$\int_0^2 \left(x-1-e^{-\frac{1}{2}x}\right)dx$

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9. Integration durch Substitution

Ausführliche Lösungen:

A1 Ausführliche Lösung

$\int \frac{3}{4x+1}\,dx$ Substitution: $u = 4x+1$ $\frac{du}{dx} = 4 \Rightarrow dx = \frac{1}{4}du$

$\frac{3}{4}\int \frac{1}{u}\,du = \frac{3}{4}\cdot \ln(u) + C$ mit $u = 4x+1$ wird

$$\int \frac{3}{4x+1}\,dx = \underline{\underline{\frac{3}{4}\cdot \ln(4x+1) + C}}$$

A2 Ausführliche Lösung

$\int_0^2 \frac{4}{4-x}\,dx$ Substitution: $u = 4-x$

$\frac{du}{dx} = -1 \Rightarrow dx = -du$ untere Grenze: $u(0) = 4$ obere Grenze: $u(2) = 4-2 = 2$

$$-4\int_4^2 \frac{1}{u}\,du = 4\int_2^4 \frac{1}{u}\,du = \left[4\cdot \ln(u)\right]_2^4 = 4\cdot \ln(4) - 4\cdot \ln(2)$$

$$= 4\left[\ln(4) - \ln(2)\right] = 4\cdot \ln\left(\frac{4}{2}\right) = 4\cdot \ln(2) \approx \underline{\underline{2,773}}$$

A3 Ausführliche Lösung

$\int \frac{2}{(1-x)^2}\,dx$ Substitution: $u = 1-x$ $\frac{du}{dx} = -1 \Rightarrow dx = -du$

$-2\int \frac{1}{u^2}\,du = -2\int u^{-2}\,du = -2\cdot \frac{1}{-1}\cdot u^{-1} = \frac{2}{u}$ mit $u = 1-x$ wird

$$\int \frac{2}{(1-x)^2}\,dx = \underline{\underline{\frac{2}{1-x} + C}}$$

A4 Ausführliche Lösung

$\int \frac{6}{(2x-1)^3}\,dx$ Substitution: $u = 2x-1$ $\frac{du}{dx} = 2 \Rightarrow dx = \frac{1}{2}du$

$\frac{6}{2}\int \frac{1}{u^3}\,du = 3\int u^{-3}\,du = 3\cdot \frac{1}{-2}\cdot u^{-2} = -\frac{3}{2}\cdot \frac{1}{u^2}$ mit $u = 2x-1$ wird

$$\int \frac{6}{(2x-1)^3}\,dx = \underline{\underline{-\frac{3}{2}\cdot \frac{1}{(2x-1)^2} + C}}$$

A5 Ausführliche Lösung

$$\int_{-2}^{2}\frac{10}{(x-4)^5}\,dx \qquad \text{Substitution: } u = x-4 \qquad \frac{du}{dx}=1 \Rightarrow dx = du$$

untere Grenze: $u(-2) = -2-4 = -6$ obere Grenze: $u(2) = 2-4 = -2$

$$10\int_{-6}^{-2}\frac{1}{u^5}\,du = 10\int_{-6}^{-2}u^{-5}\,du = \left[10\cdot\frac{1}{-4}\cdot u^{-4}\right]_{-6}^{-2} = -\frac{5}{2}\cdot\left[\frac{1}{u^4}\right]_{-6}^{-2}$$

$$= -\frac{5}{2}\left[\frac{1}{16}-\frac{1}{1296}\right] = \underline{\underline{-\frac{25}{162} \approx -0{,}154}}$$

A6 Ausführliche Lösung

$$\int_{-2}^{2}e^{1-x}\,dx \qquad \text{Substitution: } u = 1-x \qquad \frac{du}{dx}=-1 \Rightarrow dx = -du$$

untere Grenze: $u(-2) = 1-(-2) = 3$ obere Grenze: $u(2) = 1-2 = -1$

$$-\int_{3}^{-1}e^u\,du = \int_{-1}^{3}e^u\,du = \left[e^u\right]_{-1}^{3} = \underline{\underline{e^3 - e^{-1} \approx 19{,}718}}$$

A7 Ausführliche Lösung

$$\int_{0}^{4}e^{\frac{1}{2}x}\,dx \qquad \text{Substitution: } u = \frac{1}{2}x \qquad \frac{du}{dx}=\frac{1}{2} \Rightarrow dx = 2du$$

untere Grenze: $u(0) = 0$ obere Grenze: $u(4) = 2$

$$2\int_{0}^{2}e^u\,du = 2\left[e^u\right]_{0}^{2} = 2\cdot e^2 - 2\cdot e^0 = \underline{\underline{2\cdot e^2 - 2 \approx 12{,}778}}$$

A8 Ausführliche Lösung

$$\int_{1}^{2}e^{4-2x}\,dx \qquad \text{Substitution: } u = 4-2x \qquad \frac{du}{dx}=-2 \Rightarrow dx = -\frac{1}{2}du$$

untere Grenze: $u(1) = 4-2 = 2$ obere Grenze: $u(2) = 4-4 = 0$

$$-\frac{1}{2}\int_{2}^{0}e^u\,du = \frac{1}{2}\int_{0}^{2}e^u\,du = \left[\frac{1}{2}e^u\right]_{0}^{2} = \frac{1}{2}\cdot e^2 - \frac{1}{2}\cdot e^0 = \underline{\underline{\frac{1}{2}\cdot e^2 - \frac{1}{2} \approx 3{,}195}}$$

A9 Ausführliche Lösung

$$\int_{1}^{2}\frac{4}{e^{2x-4}}\,dx = 4\int_{1}^{2}e^{4-2x}\,dx \qquad \text{Substitution: } u = 4-2x \qquad \frac{du}{dx}=-2 \Rightarrow dx = -\frac{1}{2}du$$

untere Grenze: $u(1) = 4-2 = 2$ obere Grenze: $u(2) = 4-4 = 0$

$$-4\cdot\frac{1}{2}\int_{2}^{0}e^u\,du = 2\int_{0}^{2}e^u\,du = \left[2e^u\right]_{0}^{2} = 2\cdot e^2 - 2\cdot e^0 = \underline{\underline{2\cdot e^2 - 2 \approx 12{,}788}}$$

A10	Ausführliche Lösung
	$\int_0^2 \left(x-1-e^{-\frac{1}{2}x} \right) dx = \int_0^2 (x-1)\,dx - \int_0^2 e^{-\frac{1}{2}x}\,dx$
	1. Integral: $\int_0^2 (x-1)\,dx = \left[\frac{1}{2}x^2 - x \right]_0^2 = \frac{1}{2}\cdot 4 - 2 - (0) = 0$
	2. Integral: $-\int_0^2 e^{-\frac{1}{2}x}\,dx$ Substitution: $u = -\frac{1}{2}x$ $\frac{du}{dx} = -\frac{1}{2} \Rightarrow dx = -2du$
	untere Grenze: $u(0) = 0$ obere Grenze: $u(2) = -\frac{1}{2}\cdot 2 = -1$
	$= (-1)(-2)\int_0^{-1} e^u\,du = 2\int_0^{-1} e^u\,du = -2\int_{-1}^{0} e^u\,du = \left[-2e^u \right]_{-1}^{0}$
	$= -2\cdot e^0 - \left(-2\cdot e^{-1} \right) = \underline{\underline{-2+2\cdot e^{-1} \approx -1{,}264}}$