Lösungen lineare Funktionen Teil IV

Ausführliche Lösungen:

A1	Ausführliche Lösung	
	a)	$f(1) = 7 \Rightarrow P_1(1\,\|\,7)\,;\, f(-1) = 3 \Rightarrow P_1(-1\,\|\,3)$
		$a_1 = \frac{y_2 - y_1}{x_1 - x_2} = \frac{3-7}{-1-1} = 2 \Rightarrow f(x) = 2x + a_0$
		$P_1(1\,\|\,7): \quad \Rightarrow f(1) = 2 \cdot 1 + a_0 = 7 \Rightarrow a_0 = 5 \Rightarrow \underline{\underline{f(x) = 2x + 5}}$

A1	Ausführliche Lösung	
	b)	$f(a) = 0 \Rightarrow P_1(a\,\|\,0)\,;\, f(0) = a \Rightarrow P_2(0\,\|\,a) \Rightarrow a_0 = a \Rightarrow f(x) = a_1x + a$
		$a_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{a-0}{0-a} = -1 \Rightarrow \underline{\underline{f(x) = -x + a}}$

A1	Ausführliche Lösung	
	c)	$f(a) = 1 \Rightarrow P_1(a\,\|\,1)\,;\, f(2a) = -1 \Rightarrow P_1(2a\,\|\,-1)$
		$a_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-1}{2a-a} = -\frac{2}{a} \Rightarrow f(x) = -\frac{2}{a}x + a_0$
		$P_1(a\,\|\,1): \quad f(a) = -\frac{2}{a} \cdot a + a_0 = 1 \Rightarrow a_0 = 3 \Rightarrow \underline{\underline{f(x) = -\frac{2}{a}x + 3\,;\, a \neq 0}}$

A2	Ausführliche Lösung
	$P_1\left(\sqrt{k}\,\|\,k\right);\, P_2(1\,\|\,1)$ zu zeigen ist: $a_1 = \sqrt{k} + 1$ und $P_y\left(0\,\|\,-\sqrt{k}\right) \Rightarrow a_0 = -\sqrt{k}$
	$a_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-k}{1-\sqrt{k}} = \frac{(1-k)\left(1+\sqrt{k}\right)}{\left(1-\sqrt{k}\right)\left(1+\sqrt{k}\right)} = \frac{(1-k)\left(1+\sqrt{k}\right)}{(1-k)} = 1 + \sqrt{k} = \underline{\underline{\sqrt{k} + 1}}$
	$\Rightarrow f(x) = \left(\sqrt{k} + 1\right)x + a_0$
	$P_2(1\,\|\,1): \quad f(1) = \left(\sqrt{k} + 1\right) \cdot 1 + a_0 = 1 \Rightarrow \underline{\underline{a_0 = -\sqrt{k}}} \Rightarrow \underline{\underline{f(x) = \left(\sqrt{k} + 1\right)x - \sqrt{k}}}$
	$f(0) = \left(\sqrt{k} + 1\right) \cdot 0 - \sqrt{k} = -\sqrt{k} \Rightarrow P_y\left(0\,\|\,-\sqrt{k}\right)$

A3 Ausführliche Lösung

$$P\left(\frac{k}{2}\sqrt{2}\,|\,k\right) \Rightarrow P_1\left(\frac{k_1}{2}\sqrt{2}\,|\,k_1\right) \Rightarrow P_2\left(\frac{k_2}{2}\sqrt{2}\,|\,k_2\right) \Rightarrow P_3\left(\frac{k_3}{2}\sqrt{2}\,|\,k_3\right)$$

$$a_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{k_2 - k_1}{\frac{k_2}{2}\sqrt{2} - \frac{k_1}{2}\sqrt{2}} = \frac{(t_2 - t_1)}{\frac{1}{2}\sqrt{2}(k_2 - k_1)}$$

$$= \frac{1}{\frac{1}{2}\sqrt{2}} = \frac{2}{\sqrt{2}} = \frac{2\cdot\sqrt{2}}{\sqrt{2}\cdot\sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2}$$

$$\Rightarrow f(x) = \sqrt{2}x + a_0$$

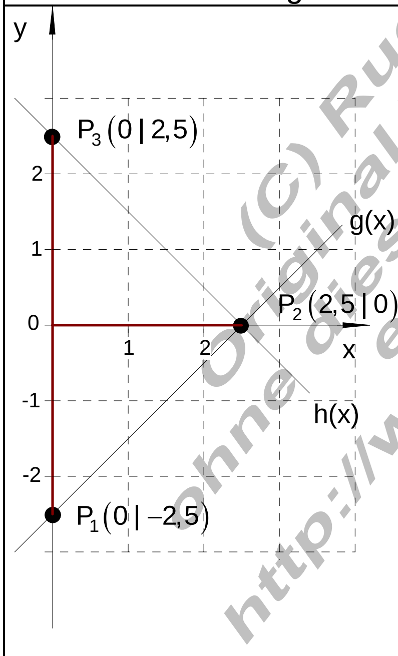
$$P_1\left(\frac{k_1}{2}\sqrt{2}\,|\,k_1\right): \quad f\left(\frac{k_1}{2}\sqrt{2}\right) = \sqrt{2}\left(\frac{k_1}{2}\sqrt{2}\right) + a_0 = k_1$$

$$\Rightarrow \sqrt{2}\left(\frac{k_1}{2}\sqrt{2}\right) + a_0 = k_1 \Leftrightarrow \sqrt{2}\cdot\sqrt{2}\cdot\frac{k_1}{2} + a_0 = k_1 \Leftrightarrow k_1 + a_0 = k_1 \Rightarrow a_0 = 0$$

$$\Rightarrow \underline{\underline{f(x) = \sqrt{2}x}}$$

$$P_3\left(\frac{k_3}{2}\sqrt{2}\,|\,k_3\right): \quad f\left(\frac{k_3}{2}\sqrt{2}\right) = \sqrt{2}\left(\frac{k_3}{2}\sqrt{2}\right) = k_3$$

A4 Ausführliche Lösung



Steigung von g(x):

$$a_{1g} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-(-2,5)}{2,5-0} = 1$$

Steigung von h(x):

$$a_{1h} = \frac{y_3 - y_2}{x_3 - x_2} = \frac{2,5-0}{0-2,5} = -1$$

Das Dreieck ist gleichschenklig für $\underline{\underline{a_1 = \pm 1}}$

A5 Ausführliche Lösung

$$f(x) = 3e^{-0,5x}$$

$$h(0) = f(0) = 3e^0 = 3 \cdot 1 = 3 \Rightarrow P_1(0\,|\,3) \Rightarrow a_0 = 3 \qquad \text{Bemerkung: } e^0 = 1$$

$$h(-2) = f(-2) = 3e^{-0,5\cdot(-2)} = 3e^1 = 3e \Rightarrow P_2(-2\,|\,3e)$$

$$a_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3e-3}{-2-0} = \frac{3e-3}{-2} = -\frac{3}{2}(e-1) \Rightarrow \underline{\underline{h(x) = -\frac{3}{2}(e-1)x + 3}}$$

A6 Ausführliche Lösung

a)

$$\underline{\underline{f(x) = \frac{2}{3}x + 2}} \qquad \underline{\underline{h(x) = -\frac{1}{4}x - 2}} \text{ direkt abgelesen}$$

$$g(x) = \frac{5}{2}x + a_0 \text{ mit } P(4\,|\,1): \quad g(4) = \frac{5}{2}\cdot 4 + a_0 = 1 \Rightarrow a_0 = -9 \Rightarrow \underline{\underline{g(x) = \frac{5}{2}x - 9}}$$

A6 Ausführliche Lösung

b)

Der Term für g(x) lässt sich aus dem Graphen ablesen: $g(x) = \frac{1}{4}x - 2$

Von f(x) ist bekannt: $P_1(2\,|\,5)$ und $P_2(3\,|\,g(3))$

$$g(3) = \frac{1}{4}\cdot 3 - 2 = -\frac{5}{4} \Rightarrow P_2\left(3\,\middle|\,-\frac{5}{4}\right)$$

$$a_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-\frac{5}{4} - 5}{3-2} = \frac{-\frac{5}{4} - \frac{20}{4}}{1} = -\frac{25}{4} \Rightarrow f(x) = -\frac{25}{4}x + a_0$$

$$P_1(2\,|\,5): \quad f(2) = -\frac{25}{4}\cdot 2 + a_0 = 5 \Rightarrow a_0 = \frac{35}{2} \Rightarrow \underline{\underline{f(x) = -\frac{25}{4}x + \frac{35}{2}}}$$

A7 Ausführliche Lösung

a)

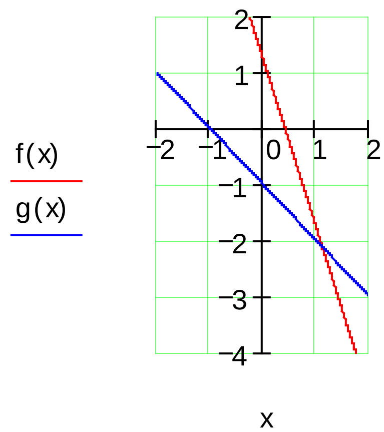
$$f(x) = -3x + \frac{5}{4};\ g(x) = -x - 1$$

$$f(x_s) = g(x_s)$$

$$\Rightarrow -3x_s + \frac{5}{4} = -x_s - 1 \Leftrightarrow x_s = \frac{9}{8}$$

$$y_s = g(x_s) = g\left(\frac{9}{8}\right) = -\frac{9}{8} - 1 = -\frac{17}{8}$$

$$\Rightarrow \underline{\underline{S\left(\frac{9}{8}\,\middle|\,-\frac{17}{8}\right)}}$$



A7 Ausführliche Lösung

b)

$f: 2y - x = 3 \Rightarrow f(x) = \frac{1}{2}x + \frac{3}{2}$

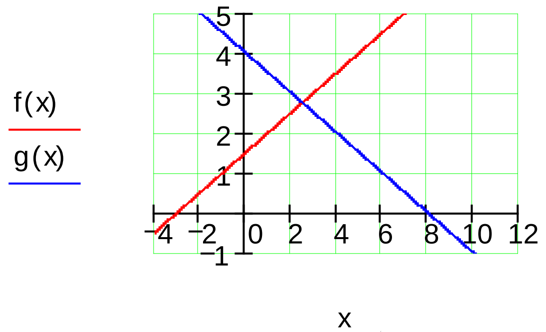
$f(x_s) = g(x_s)$

$\frac{1}{2}x_s + \frac{3}{2} = -\frac{1}{2}x_s + 4$

$\Rightarrow x_s = \frac{5}{2}$

$y_s = f(x_s) = f\left(\frac{5}{2}\right) = \frac{11}{4}$

$\Rightarrow \underline{\underline{S\left(\frac{5}{2} \middle| \frac{11}{4}\right)}}$



A7 Ausführliche Lösung

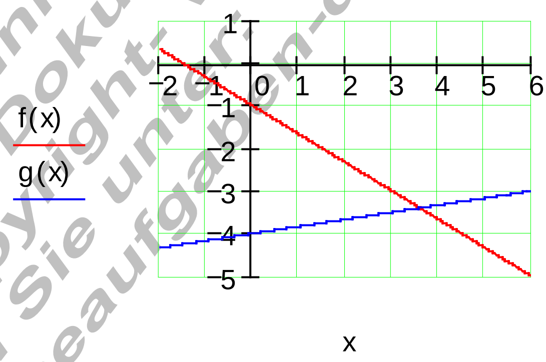
c)

$f(x_s) = g(x_s)$

$\Rightarrow -\frac{2}{3}x_s - 1 = \frac{1}{6}x_s - 4 \Rightarrow x_s = \frac{18}{5}$

$y_s = g(x_s) = g\left(\frac{18}{5}\right) = -\frac{17}{5}$

$\Rightarrow \underline{\underline{S\left(\frac{18}{5} \middle| -\frac{17}{5}\right)}}$



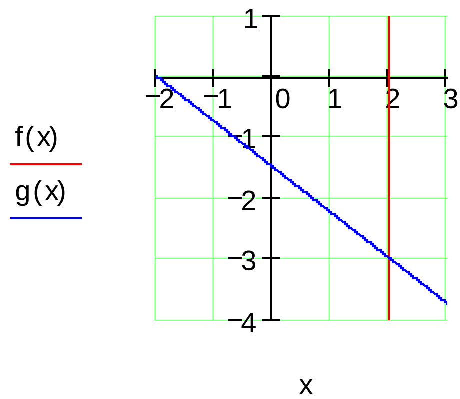
A7 Ausführliche Lösung

d)

$x = 2 \Rightarrow x_s = 2$

$y_s = g(x_s) = g(2) = -\frac{3}{4} \cdot 2 - \frac{3}{2} = -3$

$\Rightarrow \underline{\underline{S(2 \mid -3)}}$



A8 Ausführliche Lösung

Geraden durch den Nullpunkt: $f(x) = a_1 x \quad a_1 \in \mathbb{R}$

Verschoben durch $x = 4 \Rightarrow f^*(x) = a_1(x-4) \quad a_1 \in \mathbb{R}$

Beispiele: $a_1 = 1 \Rightarrow f(x) = x - 4$

$a_1 = 2 \Rightarrow g(x) = 2(x-4) = 2x - 8$