

Lösungen Potenzen X

Ergebnisse:

E1	Ergebnisse	
a)	$\frac{(x-y)(x^2+2xy+y^2)}{x^4-y^4} \cdot \frac{(x^2-y^2)(x+y)}{(x-y)(x^2+y^2)} = \frac{1}{x+y}$	
b)	$\frac{ax+a+x+1}{a^2-1} = \frac{x+1}{a-1}$	
c)	$\frac{(a^2+2ab+b^2) \cdot a^2}{a^2-b^2} : \frac{a^2}{(a+b)(a-b)} = (a+b)^2$	
d)	$\frac{am+bm-an-bn}{a^2-b^2} = \frac{m-n}{a-b}$	

E2	Ergebnisse			
a)	$\frac{ac-bc+ad-bd}{(c+d)^2} = \frac{a-b}{c+d}$	b)	$\frac{x^3+y^3}{x^2-y^2} \cdot \frac{x+y}{x^2-xy+y^2} = \frac{x+y}{x-y}$	
c)	$\left(\frac{a^5b^{-2}c^7}{d^8c^9}\right)^{10} : (a^2b^4c^5d^6)^{12} = \frac{a^{26}}{b^{68}c^{80}d^{152}}$		d)	$\frac{a^2+ab+b^2}{(a+b)^2} \cdot \frac{a^2-b^2}{a^3-b^3} = \frac{1}{a+b}$

E3	Ergebnisse		
a)	$\left[(x^2-y^2)^3\right]^4 : (x-y)^{12} = (x+y)^{12}$	b)	$(x^2-2x+1) \cdot \frac{x^2+x}{x^2-1} = x^2-x$
c)	$\frac{ax+bx-am-bm}{a^2-b^2} = \frac{x-m}{a-b}$	d)	$\left[(m+n)^{\frac{1}{2}}\right]^{-\frac{1}{2}} \cdot \left[(m+n)^{\frac{2}{3}}\right]^{\frac{3}{4}} = (m+n)^{\frac{1}{4}}$

E4	Ergebnisse		
a)	$\left[(a+b)^2\right]^3 : (a+b)^5 = a+b$	b)	$\left(\frac{a^{-2}c^4}{b^4}\right)^{-3} : \left(\frac{2a^{-3}}{b^2c^{-5}}\right)^{-4} : \left(\frac{4b^2c^3}{a^3}\right)^2 = c^2$
c)	$\frac{\left[(-x^2)^2\right]^2 \cdot x^{2a}}{x^6} = x^{2a+2}$	d)	$\left(\frac{2a^3b^6}{c^6}\right)^2 : \left(\frac{c^4d^{-4}}{4a^4b^5}\right)^{-3} : \left(\frac{4a^3d^6}{b^{-2}}\right)^{-2} = b$

E5	Ergebnisse	
a)	$\left(\frac{7a^2b^3c^4}{8x^5y^7z^7}\right)^u : \left(\frac{21a^2b^2c^4}{16x^6y^7z^8}\right)^u = \left(\frac{2bxz}{3}\right)^u$	
b)	$\left[(a^2-x^2)^{-3}\right]^{-2} \cdot [(a+x)(a-x)]^{-5} = a^2-x^2$	

E6 Ergebnisse			
a)	$\left[(x^2 - y^2)^3 \right]^4 : (x - y)^{12} = (x + y)^{12}$	b)	$\left(\frac{7xy^2}{3p^2q^2} \right)^5 \cdot \left(\frac{18p^2q^2}{14xy^2} \right)^5 = 243$
c)	$\left(\frac{4m^7n^8p^5}{28rs^2} \right)^x : \left(\frac{m^7n^7p^6}{35r^2s^2} \right)^x = \left(\frac{5nr}{p} \right)^x$	d)	$\left(\frac{a^2}{x^3} \right)^{-2} \cdot \left(\frac{2x^2}{5a^3} \right)^{-1} \cdot 2ax^{-4} = 5$

Potenzgesetze

$a^m \cdot a^n = a^{m+n}$	$\frac{a^m}{a^n} = a^{m-n}$	$a^n \cdot b^n = (a \cdot b)^n$	$\frac{a^n}{b^n} = \left(\frac{a}{b} \right)^n$
$(a^n)^m = a^{n \cdot m}$	$\sqrt[n]{a^m} = a^{\frac{m}{n}}$	$a^0 = 1$	$\frac{1}{a^n} = a^{-n}$

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Ausführliche Lösungen :

A1	Ausführliche Lösung
	<p>a) $\frac{(x-y)(x^2+2xy+y^2)}{x^4-y^4} \cdot \frac{(x^2-y^2)(x+y)}{(x-y)(x^2+y^2)} = \frac{(x-y)(x+y)^2 \cdot (x-y)(x^2+y^2)}{(x^4-y^4)(x^2-y^2)(x+y)}$ $= \frac{(x-y)^2(x+y)^2(x^2+y^2)}{(x^2-y^2)(x^2+y^2)(x-y)(x+y)(x+y)}$ $= \frac{(x-y)^2 \cancel{(x+y)^2} \cancel{(x^2+y^2)}}{(x^2-y^2) \cancel{(x^2+y^2)} (x-y) \cancel{(x+y)} \cancel{(x+y)}}$ $= \frac{(x-y)^2}{(x-y)(x+y)(x-y)}$ $= \frac{\cancel{(x-y)}^2}{\cancel{(x-y)} \cancel{(x-y)} (x+y)} = \underline{\underline{\frac{1}{x+y}}}$ </p>

A1	Ausführliche Lösung
	<p>b) $\frac{ax+a+x+1}{a^2-1} = \frac{a(x+1)+1(x+1)}{(a-1)(a+1)}$ $= \frac{(x+1)(a+1)}{(a-1)(a+1)}$ $= \frac{\cancel{(x+1)} \cancel{(a+1)}}{\cancel{(a-1)} \cancel{(a+1)}}$ $= \underline{\underline{\frac{x+1}{a-1}}}$ </p>

A1	Ausführliche Lösung
c)	$\frac{(a^2 + 2ab + b^2) \cdot a^2}{a^2 - b^2} : \frac{a^2}{(a+b)(a-b)} = \frac{(a+b)^2 \cdot a^2 (a+b)(a-b)}{(a^2 - b^2) \cdot a^2}$ $= \frac{(a+b)^2 \cdot \cancel{a^2} (a+b)(a-b)}{(a^2 - b^2) \cdot \cancel{a^2}}$ $= \frac{(a+b)^2 (a^2 - b^2)}{(a^2 - b^2)}$ $= \frac{(a+b)^2 \cancel{(a^2 - b^2)}}{\cancel{(a^2 - b^2)}}$ $= \underline{\underline{(a+b)^2}}$

A1	Ausführliche Lösung
d)	$\frac{am + bm - an - bn}{a^2 - b^2} = \frac{m(a+b) - n(a+b)}{(a-b)(a+b)}$ $= \frac{(a+b)(m-n)}{(a-b)(a+b)}$ $= \frac{\cancel{(a+b)}(m-n)}{\cancel{(a+b)}(a-b)}$ $= \underline{\underline{\frac{m-n}{a-b}}}$

A2	Ausführliche Lösung
a)	$\frac{ac - bc + ad - bd}{(c+d)^2} = \frac{c(a-b) + d(a-b)}{(c+d)^2}$ $= \frac{(a-b)(c+d)}{(c+d)(c+d)}$ $= \frac{(a-b) \cancel{(c+d)}}{\cancel{(c+d)}(c+d)}$ $= \underline{\underline{\frac{a-b}{c+d}}}$

A2	Ausführliche Lösung	
	b)	$\frac{x^3 + y^3}{x^2 - y^2} \cdot \frac{x + y}{x^2 - xy + y^2} = \frac{(x^3 + y^3)(x + y)}{(x^2 - y^2)(x^2 - xy + y^2)}$ $= \frac{(x^3 + y^3)(x + y)}{(x - y)(x + y) \cdot (x^2 - xy + y^2)}$ $= \frac{(x^3 + y^3)(x + y)}{(x - y)(x^3 - x^2y + xy^2 + x^2y - xy^2 + y^3)}$ $= \frac{(x^3 + y^3)(x + y)}{(x - y)(x^3 - \cancel{x^2y} + \cancel{xy^2} + \cancel{x^2y} - \cancel{xy^2} + y^3)}$ $= \frac{(x^3 + y^3)(x + y)}{(x - y)(x^3 + y^3)}$ $= \frac{\cancel{(x^3 + y^3)}(x + y)}{(x - y)\cancel{(x^3 + y^3)}}$ $= \frac{x + y}{x - y}$

A2	Ausführliche Lösung	
	c)	$\left(\frac{a^5 b^{-2} c^7}{d^8 c^9}\right)^{10} : (a^2 b^4 c^5 d^6)^{12} = \frac{a^{50} b^{-20} c^{70}}{d^{80} c^{90}} \cdot \frac{a^{24} b^{48} c^{60} d^{72}}{1}$ $= \frac{a^{50} b^{-20} c^{70} \cdot 1}{d^{80} c^{90} \cdot a^{24} b^{48} c^{60} d^{72}}$ $= \frac{a^{50} c^{70}}{a^{24} b^{48} b^{20} c^{90+60} c^{60} d^{80} d^{72}}$ $= \frac{a^{50-24}}{b^{48+20} c^{90+60-70} d^{80+72}}$ $= \frac{a^{26}}{b^{68} c^{80} d^{152}}$

A2	<p>Ausführliche Lösung</p> <p>d)</p> $\frac{a^2 + ab + b^2}{(a+b)^2} \cdot \frac{a^2 - b^2}{a^3 - b^3} = \frac{(a^2 + ab + b^2)(a^2 - b^2)}{(a+b)^2(a^3 - b^3)}$ $= \frac{(a^2 + ab + b^2)(a-b)(a+b)}{(a+b)(a+b)(a^3 - b^3)}$ $= \frac{(a^2 + ab + b^2)(a-b) \cancel{(a+b)}}{\cancel{(a+b)}(a+b)(a^3 - b^3)}$ $= \frac{a^3 - a^2b + a^2b - ab^2 + ab^2 - b^3}{(a+b)(a^3 - b^3)}$ $= \frac{a^3 - \cancel{a^2b} + \cancel{a^2b} - \cancel{ab^2} + \cancel{ab^2} - b^3}{(a+b)(a^3 - b^3)}$ $= \frac{a^3 - b^3}{(a+b)(a^3 - b^3)}$ $= \frac{\cancel{(a^3 - b^3)}}{(a+b) \cancel{(a^3 - b^3)}}$ $= \frac{1}{\underline{\underline{a+b}}}$
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A3	<p>Ausführliche Lösung</p> <p>a)</p> $\left[(x^2 - y^2)^3 \right]^4 : (x-y)^{12} = \frac{\left[(x-y)^3 (x+y)^3 \right]^4}{(x-y)^{12}}$ $= \frac{(x-y)^{12} (x+y)^{12}}{(x-y)^{12}}$ $= \frac{\cancel{(x-y)^{12}} (x+y)^{12}}{\cancel{(x-y)^{12}}}$ $= \underline{\underline{(x+y)^{12}}}$
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A3	Ausführliche Lösung
b)	$\begin{aligned} (x^2 - 2x + 1) \cdot \frac{x^2 + x}{x^2 - 1} &= \frac{(x-1)^2 \cdot x(x+1)}{(x-1)(x+1)} \\ &= \frac{(x-1)(x-1) \cdot x(x+1)}{(x-1)(x+1)} \\ &= \frac{\cancel{(x-1)}(x-1) \cdot x \cancel{(x+1)}}{\cancel{(x-1)}(x+1)} \\ &= \underline{\underline{x(x-1) = x^2 - x}} \end{aligned}$

A3	Ausführliche Lösung
c)	$\begin{aligned} \frac{ax + bx - am - bm}{a^2 - b^2} &= \frac{x(a+b) - m(a+b)}{(a-b)(a+b)} \\ &= \frac{(a+b)(x-m)}{(a-b)(a+b)} \\ &= \frac{\cancel{(a+b)}(x-m)}{(a-b)\cancel{(a+b)}} \\ &= \underline{\underline{\frac{x-m}{a-b}}} \end{aligned}$

A3	Ausführliche Lösung
d)	$\begin{aligned} \left[(m+n)^{\frac{1}{2}} \right]^{-\frac{1}{2}} \cdot \left[(m+n)^{\frac{2}{3}} \right]^{\frac{3}{4}} &= (m+n)^{\frac{1}{2} \cdot \left(-\frac{1}{2}\right)} \cdot (m+n)^{\frac{2}{3} \cdot \frac{3}{4}} \\ &= (m+n)^{-\frac{1}{4}} \cdot (m+n)^{\frac{1}{2}} \\ &= (m+n)^{-\frac{1}{4} + \frac{1}{2}} \\ &= \underline{\underline{(m+n)^{\frac{1}{4}}}} \end{aligned}$

A4	Ausführliche Lösung
a)	$\left[(a+b)^2 \right]^3 : (a+b)^5 = \frac{(a+b)^6}{(a+b)^5} = \underline{\underline{a+b}}$

A4	Ausführliche Lösung
b)	$\begin{aligned} \left(\frac{a^{-2}c^4}{b^4} \right)^{-3} : \left(\frac{2a^{-3}}{b^2c^{-5}} \right)^{-4} : \left(\frac{4b^2c^3}{a^3} \right)^2 &= \left(\frac{b^4}{a^{-2}c^4} \right)^3 : \left(\frac{b^2c^{-5}}{2a^{-3}} \right)^4 : \left(\frac{4b^2c^3}{a^3} \right)^2 \\ &= \frac{b^{12}}{a^{-6}c^{12}} : \frac{b^8c^{-20}}{2^4a^{-12}} : \frac{4^2b^4c^6}{a^6} \\ &= \frac{b^{12} \cdot 2^4a^{-12}}{a^{-6}c^{12} \cdot b^8c^{-20}} : \frac{2^4b^4c^6}{a^6} \\ &= \frac{b^{12} \cdot 2^4a^{-12} \cdot a^{-6}}{a^{-6}c^{12} \cdot b^8c^{-20} \cdot 2^4b^4c^6} \\ &= \frac{2^4 \cdot a^{-6}b^{12}c^{-2}}{2^4 \cdot a^{-6}b^{12}c^{-2}} \\ &= \frac{\cancel{2^4} \cdot \cancel{a^{-6}}b^{12}}{\cancel{2^4} \cdot \cancel{a^{-6}}b^{12}c^{-2}} = \frac{1}{c^{-2}} = \underline{\underline{c^2}} \end{aligned}$

A4	Ausführliche Lösung
c)	$\frac{\left[(-x^2)^2 \right]^2 \cdot x^{2a}}{x^6} = \frac{\left[x^4 \right]^2 \cdot x^{2a}}{x^6} = \frac{x^8 \cdot x^{2a}}{x^6} = x^2 \cdot x^{2a} = \underline{\underline{x^{2a+2}}}$

A4	Ausführliche Lösung
d)	$\begin{aligned} \left(\frac{2a^3b^6}{c^6} \right)^2 : \left(\frac{c^4d^{-4}}{4a^4b^5} \right)^{-3} : \left(\frac{4a^3d^6}{b^{-2}} \right)^{-2} &= \left(\frac{2a^3b^6}{c^6} \right)^2 : \left(\frac{4a^4b^5}{c^4d^{-4}} \right)^3 : \left(\frac{b^{-2}}{4a^3d^6} \right)^2 \\ &= \frac{2^2a^6b^{12}}{c^{12}} : \frac{2^6a^{12}b^{15}}{c^{12}d^{-12}} : \frac{b^{-4}}{2^4a^6d^{12}} \\ &= \frac{2^2a^6b^{12} \cdot c^{12}d^{-12}}{c^{12} \cdot 2^6a^{12}b^{15}} : \frac{1}{2^4a^6b^4d^{12}} \\ &= \frac{2^2 \cdot 2^4 \cdot a^6 \cdot a^6 \cdot b^{12} \cdot b^4 \cdot c^{12} \cdot d^{-12} \cdot d^{12}}{c^{12} \cdot 2^6 a^{12} b^{15}} \\ &= \frac{2^6 \cdot a^{12} \cdot b^{16} \cdot c^{12} \cdot d^0}{2^6 \cdot a^{12} \cdot b^{15} \cdot c^{12}} \\ &= \frac{\cancel{2^6} \cdot \cancel{a^{12}} \cdot b^{16} \cdot \cancel{c^{12}} \cdot 1}{\cancel{2^6} \cdot \cancel{a^{12}} \cdot b^{15} \cdot \cancel{c^{12}}} \\ &= \frac{b^{16}}{b^{15}} = \underline{\underline{b}} \end{aligned}$

A5	Ausführliche Lösung	
	a)	$\left(\frac{7a^2b^3c^4}{8x^5y^7z^7}\right)^u \cdot \left(\frac{21a^2b^2c^4}{16x^6y^7z^8}\right)^u = \frac{(7a^2b^3c^4)^u (16x^6y^7z^8)^u}{(8x^5y^7z^7)^u (21a^2b^2c^4)^u}$ $= \frac{[(7a^2b^3c^4)(16x^6y^7z^8)]^u}{[(8x^5y^7z^7)(21a^2b^2c^4)]^u}$ $= \left[\frac{7a^2b^3c^4 \cdot 16x^6y^7z^8}{8x^5y^7z^7 \cdot 21a^2b^2c^4}\right]^u$ $= \left[\frac{7 \cdot 2 \cdot 8a^2b^3c^4x^6y^7z^8}{7 \cdot 3 \cdot 8a^2b^2c^4x^5y^7z^7}\right]^u$ $= \left[\frac{\cancel{7} \cdot 2 \cdot \cancel{8}a^{\cancel{2}}b^3\cancel{c^4}x^6y^{\cancel{7}}z^8}{\cancel{7} \cdot 3 \cdot \cancel{8}a^{\cancel{2}}b^2\cancel{c^4}x^5y^{\cancel{7}}z^7}\right]^u$ $= \left(\frac{2b^3x^6z^8}{3b^2x^5z^7}\right)^u = \underline{\underline{\left(\frac{2bxz}{3}\right)^u}}$

A5	Ausführliche Lösung	
	b)	$\left[(a^2 - x^2)^{-3}\right]^{-2} \cdot [(a+x)(a-x)]^{-5} = (a^2 - x^2)^6 \cdot [a^2 - x^2]^{-5}$ $= \frac{(a^2 - x^2)^6}{(a^2 - x^2)^5}$ $= \underline{\underline{a^2 - x^2}}$

A6	Ausführliche Lösung	
	a)	$\left[(x^2 - y^2)^3\right]^4 : (x-y)^{12} = \frac{(x^2 - y^2)^{12}}{(x-y)^{12}}$ $= \frac{[(x-y)(x+y)]^{12}}{(x-y)^{12}}$ $= \left[\frac{(x-y)(x+y)}{(x-y)}\right]^{12}$ $= \left[\frac{\cancel{(x-y)}(x+y)}{\cancel{(x-y)}}\right]^{12}$ $= \underline{\underline{(x+y)^{12}}}$

A6	Ausführliche Lösung
b)	$\left(\frac{7xy^2}{3p^2q^2}\right)^5 \cdot \left(\frac{18p^2q^2}{14xy^2}\right)^5 = \left[\frac{7xy^2}{3p^2q^2} \cdot \frac{18p^2q^2}{14xy^2}\right]^5$ $= \left[\frac{7 \cdot 3 \cdot 6xy^2p^2q^2}{3 \cdot 2 \cdot 7p^2q^2xy^2}\right]^5$ $= \left[\frac{\cancel{7} \cdot \cancel{3} \cdot 6xy^2p^2q^2}{\cancel{3} \cdot 2 \cdot \cancel{7}p^2q^2xy^2}\right]^5$ $= \underline{\underline{3^5 = 243}}$

A6	Ausführliche Lösung
c)	$\left(\frac{4m^7n^8p^5}{28rs^2}\right)^x : \left(\frac{m^7n^7p^6}{35r^2s^2}\right)^x = \left(\frac{4m^7n^8p^5 \cdot 35r^2s^2}{28rs^2 \cdot m^7n^7p^6}\right)^x$ $= \left(\frac{4 \cdot 5 \cdot 7m^7n^8p^5r^2s^2}{4 \cdot 7rs^2m^7n^7p^6}\right)^x$ $= \left(\frac{\cancel{4} \cdot \cancel{5} \cdot \cancel{7}m^7n^8p^5r^2s^2}{\cancel{4} \cdot \cancel{7}rs^2m^7n^7p^6}\right)^x$ $= \left(\frac{5n^8p^5r^2}{m^7p^6}\right)^x = \underline{\underline{\left(\frac{5nr}{p}\right)^x}}$

A6	Ausführliche Lösung
d)	$\left(\frac{a^2}{x^3}\right)^{-2} \cdot \left(\frac{2x^2}{5a^3}\right)^{-1} \cdot 2ax^{-4} = \left(\frac{x^3}{a^2}\right)^2 \cdot \left(\frac{5a^3}{2x^2}\right)^1 \cdot \frac{2a}{x^4}$ $= \frac{x^6}{a^4} \cdot \frac{5a^3}{2x^2} \cdot \frac{2a}{x^4}$ $= \frac{x^6}{a^4} \cdot \frac{5a^3}{\cancel{2}x^2} \cdot \frac{\cancel{2}a}{x^4}$ $= \frac{5a^4x^6}{a^4x^6} = \frac{5a^4x^6}{\cancel{a^4x^6}} = \underline{\underline{5}}$