Lösungen Potenzen VI

Ergebnisse:

E1	Ergebnisse		
a)	$\frac{x^4-x^3}{x^2-x}=x^2$	b)	$\frac{\left(3x^2-6x^3\right)^2}{9x^4}=\left(1-2x\right)^2$
c)	$\frac{5x+15}{x+3}=5$	d)	$\frac{a^3+2a^2b+ab^2}{\left(a+b\right)^2}=a$
e)	$\frac{a^4-a^2b^2}{ab-a^2}=-a\left(a+b\right)$	f)	$\frac{e^{3x}+e^{2x}}{e^{2x}}=e^x+1$

E2	Ergebnisse		
a)	$\frac{x^4-6x^3}{5x-30}=\frac{1}{5}x^3$	b)	$\frac{k^3+6k^2+9k}{k^2-9}=\frac{k\left(k+3\right)}{k-3}$
c)	$\frac{a^7b^3-ab^7}{a^5b-a^2b^4}=\frac{a^6b^2-b^6}{a^4-ab^3}$	d)	$\frac{x^{2n}-10x^n+25}{x^{2n}-25}=\frac{x^n-5}{x^n+5}$
e)	$\frac{x^6-k^2}{x^4+kx}=\frac{x^3-k}{x}$	f)	$\frac{x^{n+3}-x^{n+1}}{x^{n+1}+x^n}=x\left(x-1\right)$

E3	Ergebnisse
a)	$\frac{\left(x^2+8xy+16y^2\right)}{\left(2x-3y\right)^{-2}}:\frac{x^2-16y^2}{2x-3y}=\frac{x+4y}{x-4y}\left(2x-3y\right)^3$
b)	$\frac{4k^2-4}{k^2+2k+1}=\frac{4\left(k-1\right)}{k+1}$
c)	$\frac{x^{n-1}-x^n}{x^n-x^{n+2}}=\frac{1}{x\left(x+1\right)}$
d)	$\frac{3+6x^2}{2x}-\frac{6x^3-5}{3x^2}-\frac{2x^4-2}{2x^3}=\frac{9x^2+10x+6}{6x^3}$
e)	$\frac{32}{2^{n+5}}+\frac{2^{-n+3}}{8}=2^{1-n}$
f)	$\frac{2\left(a^2+b^2\right)^2}{a^5-ab^4}=\frac{2\left(a^2+b^2\right)}{a\left(a^2-b^2\right)}$

E4	Ergebnisse
a)	$\frac{x^4 - x^3}{x^4 - x^2} = \frac{x}{x+1}$
b)	$\frac{x^3y - xy^5}{x^3y^2 - x^2y^4} = \frac{x+y^2}{xy}$
c)	$\frac{am - an + bm - bn}{a^2 - b^2} = \frac{m-n}{a-b}$

E5	Ergebnisse
a)	$y = \frac{1}{4}x^4 - 2kx^3 + \frac{9}{2}k^2x^2$ für $x = 3k \Rightarrow y = \frac{27k^4}{4}$
b)	$y = \frac{kx^3}{2(x+k)^2}$ für $x = -3k \Rightarrow y = -\frac{27k^2}{8}$
c)	$y = \frac{3}{2k^2}x^4 - \frac{4}{k}x^3 + 3x^2 - 4$ für $x = \frac{1}{3}k \Rightarrow y = \frac{11k^2}{54} - 4$
d)	$y = \frac{e^{3kx} + 4e^3}{kx - 4}$ für $x = -\frac{1}{k} \Rightarrow y = -\frac{e^{-3} + 4e^3}{5}$
e)	$y = e^{x^2 - k^2} + 3e^{5k-(k-x)}$ für $x = -k \Rightarrow y = 1 + 3e^{3k}$
f)	$y = \frac{x^3 - kx + 1}{x^3}$ für $x = \frac{3}{2k} \Rightarrow 1 - \frac{4k^3}{27}$

E6	Ergebnisse
a)	$a^n + a^{4-n} + a^{2n} = a^{2n}\left(a^{-n} + a^{4-3n} + 1\right)$
b)	$a^{k-2} + a^{3k} + a^{-k-1} = a^{-k}\left(a^{2k-2} + a^{4k} + a^{-1}\right)$
c)	$a^3 + a^{1-n} + a^{n+4} = a^{n+3}\left(a^{-n} + a^{-2n-2} + a\right)$
d)	$\frac{3}{2}x^4 + \frac{3}{4}x^3 + \frac{1}{8}x^2 = \frac{1}{8}x^2\left(12x^2 + 6x + 1\right)$
e)	$e^{3x} - 2e^{-x} = e^{-x}\left(e^{4x} - 2\right)$
f)	$ke^{2x} - 2e^{x+1} = e^x\left(ke^x - 2e\right)$

Potenzgesetze

$a^m \cdot a^n = a^{m+n}$	$\frac{a^m}{a^n} = a^{m-n}$	$a^n \cdot b^n = (a \cdot b)^n$	$\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n$
$\left(a^n\right)^m = a^{n \cdot m}$	$\sqrt[n]{a^m} = a^{\frac{m}{n}}$	$a^0 = 1$	$\frac{1}{a^n} = a^{-n}$

Ausführliche Lösungen :

A1	Ausführliche Lösungen		
a)	$\frac{x^4-x^3}{x^2-x}=\frac{x^3(x-1)}{x(x-1)}=\frac{x^3\cancel{(x-1)}}{x\cancel{(x-1)}}=\frac{x^3}{x}=\underline{\underline{x^2}}$	b)	$\frac{(3x^2-6x^3)^2}{9x^4}=\frac{[3x^2(1-2x)]^2}{9x^4}=\frac{9x^4(1-2x)^2}{9x^4}=\frac{\cancel{9x^4}(1-2x)^2}{\cancel{9x^4}}=\underline{\underline{(1-2x)^2}}$

A1	Ausführliche Lösungen		
c)	$\frac{5x+15}{x+3}=\frac{5(x+3)}{(x+3)}=\frac{5\cancel{(x+3)}}{\cancel{(x+3)}}=\underline{\underline{5}}$	d)	$\frac{a^3+2a^2b+ab^2}{(a+b)^2}=\frac{a(a^2+2ab+b^2)}{(a+b)^2}=\frac{a\cancel{(a+b)^2}}{\cancel{(a+b)^2}}=\frac{a(a+b)^2}{(a+b)^2}=\underline{\underline{a}}$

A1	Ausführliche Lösungen		
e)	$\frac{a^4-a^2b^2}{ab-a^2}=\frac{a^2(a^2-b^2)}{a(b-a)}=\frac{a(a-b)(a+b)}{-(a-b)}=\frac{a\cancel{(a-b)}(a+b)}{-1\cancel{(a-b)}}=\underline{\underline{-a(a+b)}}$	f)	$\frac{e^{3x}+e^{2x}}{e^{2x}}=\frac{e^{3x}}{e^{2x}}+\frac{e^{2x}}{e^{2x}}=e^{3x-2x}+e^{2x-2x}=e^x+\underbrace{e^0}_{1}=\underline{\underline{e^x+1}}$

A2	Ausführliche Lösungen		
a)	$\frac{x^4-6x^3}{5x-30}=\frac{x^3(x-6)}{5(x-6)}=\frac{x^3\cancel{(x-6)}}{5\cancel{(x-6)}}=\underline{\underline{\frac{1}{5}x^3}}$	b)	$\frac{k^3+6k^2+9k}{k^2-9}=\frac{k(k^2+6k+9)}{(k-3)(k+3)}=\frac{k(k+3)^2}{(k-3)(k+3)}=\underline{\underline{\frac{k(k+3)}{k-3}}}$

A2 Ausführliche Lösungen

c)
$$\frac{a^7b^3-ab^7}{a^5b-a^2b^4}=\frac{ab\left(a^6b^2-b^6\right)}{ab\left(a^4-ab^3\right)}=\underline{\underline{\frac{a^6b^2-b^6}{a^4-ab^3}}}$$

d)
$$\frac{x^{2n}-10x^n+25}{x^{2n}-25}=\frac{\left(x^n-5\right)^2}{\left(x^n-5\right)\left(x^n+5\right)}=\underline{\underline{\frac{x^n-5}{x^n+5}}}$$

A2 Ausführliche Lösungen

e)
$$\frac{x^6-k^2}{x^4+kx}=\frac{\left(x^3-k\right)\left(x^3+k\right)}{x\left(x^3+k\right)}=\underline{\underline{\frac{x^3-k}{x}}}$$

f)
$$\frac{x^{n+3}-x^{n+1}}{x^{n+1}+x^n}=\frac{x^n\cdot x\left(x^2-1\right)}{x^n\left(x+1\right)}=\frac{x\left(x-1\right)\left(x+1\right)}{\left(x+1\right)}=\underline{\underline{x\left(x-1\right)}}$$

A3 Ausführliche Lösungen

a)
$$\frac{\left(x^2+8xy+16y^2\right)}{\left(2x-3y\right)^{-2}}:\frac{x^2-16y^2}{2x-3y}$$
$$=\frac{\left(x^2+8xy+16y^2\right)\left(2x-3y\right)}{\left(2x-3y\right)^{-2}\left(x^2-16y^2\right)}$$
$$=\frac{\left(x+4y\right)^2\left(2x-3y\right)^3}{\left(x-4\right)\left(x+4\right)}$$
$$=\underline{\underline{\frac{x+4y}{x-4y}\left(2x-3y\right)^3}}$$

b)
$$\frac{4k^2-4}{k^2+2k+1}=\frac{4\left(k^2-1\right)}{\left(k+1\right)^2}=\frac{4\left(k-1\right)\left(k+1\right)}{\left(k+1\right)\left(k+1\right)}=\frac{4\left(k-1\right)\cancel{\left(k+1\right)}}{\left(k+1\right)\cancel{\left(k+1\right)}}=\underline{\underline{\frac{4\left(k-1\right)}{k+1}}}$$

A3 Ausführliche Lösungen

c)
$$\frac{x^{n-1}-x^n}{x^n-x^{n+2}}=\frac{x^{n-1}\left(1-x\right)}{x^n\left(1-x^2\right)}=\frac{1}{x}\cdot\frac{\left(1-x\right)}{\left(1-x\right)\left(1+x\right)}=\frac{1}{x}\cdot\frac{\cancel{\left(1-x\right)}}{\cancel{\left(1-x\right)}\left(1+x\right)}=\underline{\underline{\frac{1}{x\left(x+1\right)}}}$$

d)
$$\frac{3+6x^2}{2x}-\frac{6x^3-5}{3x^2}-\frac{2x^4-2}{2x^3}\quad HN=6x^3$$
$$\Rightarrow\frac{3x^2\left(3+6x^2\right)}{6x^3}-\frac{2x\left(6x^3-5\right)}{6x^3}-\frac{3\left(2x^4-2\right)}{6x^3}$$
$$=\frac{9x^2+18x^4-12x^4+10x-6x^4+6}{6x^3}$$
$$=\underline{\underline{\frac{9x^2+10x+6}{6x^3}}}$$

A3 Ausführliche Lösungen

e)

$$\frac{32}{2^{n+5}}+\frac{2^{-n+3}}{8}=\frac{2^5}{2^{n+5}}+\frac{2^{-n+3}}{2^3}$$
$$=2^{5-(n+5)}+2^{-n+3-3}$$
$$=2^{5-n-5}+2^{-n}$$
$$=2^{-n}+2^{-n}$$
$$=2\cdot 2^{-n}=\underline{\underline{2^{1-n}}}$$

f)

$$\frac{2\left(a^2+b^2\right)^2}{a^5-ab^4}=\frac{2\left(a^2+b^2\right)^2}{a\left(a^4-b^4\right)}$$
$$=\frac{2\left(a^2+b^2\right)^2}{a\left(a^2-b^2\right)\left(a^2+b^2\right)}$$
$$=\underline{\underline{\frac{2\left(a^2+b^2\right)}{a\left(a^2-b^2\right)}}}$$

A4 Ausführliche Lösungen

a)

$$\frac{x^4-x^3}{x^4-x^2}=\frac{x^3\left(x-1\right)}{x^2\left(x^2-1\right)}=\frac{x\left(x-1\right)}{\left(x-1\right)\left(x+1\right)}=\frac{x\cancel{\left(x-1\right)}}{\cancel{\left(x-1\right)}\left(x+1\right)}=\underline{\underline{\frac{x}{x+1}}}$$

A4 Ausführliche Lösungen

b)

$$\frac{x^3y-xy^5}{x^3y^2-x^2y^4}=\frac{xy\left(x^2-y^4\right)}{x^2y^2}=\frac{1}{xy}\cdot\frac{\left(x-y^2\right)\left(x+y^2\right)}{\left(x-y^2\right)}=\frac{1}{xy}\cdot\frac{\cancel{\left(x-y^2\right)}\left(x+y^2\right)}{\cancel{\left(x-y^2\right)}}=\underline{\underline{\frac{x+y^2}{xy}}}$$

A4 Ausführliche Lösungen

c)

$$\frac{am-an+bm-bn}{a^2-b^2}=\frac{a(m-n)+b(m-n)}{(a-b)(a+b)}=\frac{(m-n)(a+b)}{(a-b)(a+b)}=\underline{\underline{\frac{m-n}{a-b}}}$$

A5 Ausführliche Lösungen

a)

$$y=\frac{1}{4}x^4-2kx^3+\frac{9}{2}k^2x^2$$

für $x=3k$ gilt:

$$y=\frac{1}{4}(3k)^4-2k(3k)^3+\frac{9}{2}k^2(3k)^2$$
$$=\frac{1}{4}\cdot 3^4\cdot k^4-2\cdot 3^3\cdot k^4+\frac{3^2\cdot 3^2}{2}k$$
$$=\left(\frac{1}{4}\cdot 3^4-2\cdot 3^3+\frac{1}{2}\cdot 3^4\right)k^4$$
$$=3^3\left(\frac{1}{4}\cdot 3-2+\frac{1}{2}\cdot 3\right)k^4$$
$$=27\left(\frac{3}{4}-\frac{8}{4}+\frac{6}{4}\right)k^4$$
$$=27\cdot\left(\frac{1}{4}\right)k^4=\underline{\underline{\frac{27k^4}{4}}}$$

b)

$$y=\frac{kx^3}{2\left(x+k\right)^2}$$

für $x=-3k$ gilt:

$$y=\frac{k\left(-3k\right)^3}{2\left(-3k+k\right)^2}$$
$$=\frac{-27k^4}{2\cdot\left(-2k\right)^2}$$
$$=\frac{-27k^4}{2\cdot 4k^2}$$
$$=\underline{\underline{-\frac{27}{8}k^2}}$$

A5 Ausführliche Lösungen

c)

$$y=\frac{3}{2k^2}x^4-\frac{4}{k}x^3+3x^2-4$$

für $x=\frac{1}{3}k=\frac{k}{3}$ gilt:

$$y=\frac{3}{2k^2}\left(\frac{k}{3}\right)^4-\frac{4}{k}\left(\frac{k}{3}\right)^3+3\left(\frac{k}{3}\right)^2-4$$

$$=\frac{3\cdot k^4}{2\cdot 3^4k^2}-\frac{4k^3}{k\cdot 3^3}+3\cdot\frac{k^2}{3^2}-4$$

$$=\frac{1}{54}k^2-\frac{4}{27}k^2+\frac{1}{3}k^2-4$$

$$=\frac{1}{54}k-\frac{8}{54}k^2+\frac{18}{54}k^2-4$$

$$=\underline{\underline{\frac{11k^2}{54}-4}}$$

d)

$$y=\frac{e^{3kx}+4e^3}{kx-4}$$

für $x=-\frac{1}{k}$ gilt:

$$y=\frac{e^{3k\left(-\frac{1}{k}\right)}+4e^3}{k\left(-\frac{1}{k}\right)-4}$$

$$=\frac{e^{-3}+4e^3}{-1-4}$$

$$=\frac{e^{-3}+4e^3}{-5}$$

$$=\underline{\underline{-\frac{e^{-3}+4e^3}{5}}}$$

A5 Ausführliche Lösungen

e)

$$y=e^{x^2-k^2}+3e^{5k-(k-x)}$$

für $x=-k$ gilt:

$$y=e^{(-k)^2-k^2}+3e^{5k-(k-(-k))}$$

$$=e^{k^2-k^2}+3e^{5k-(k+k)}$$

$$=\underbrace{e^0}_{1}+3e^{5k-2k}$$

$$=\underline{\underline{1+3e^{3k}}}$$

f)

$$y=\frac{x^3-kx+1}{x^3}$$

für $x=\frac{3}{2k}$ gilt:

$$y=\frac{\left(\frac{3}{2k}\right)^3-k\left(\frac{3}{2k}\right)+1}{\left(\frac{3}{2k}\right)^3}=\frac{\frac{27}{8k^3}-\frac{3}{2}+1}{\frac{27}{8k^3}}=\frac{\frac{27}{8k^3}-\frac{1}{2}}{\frac{27}{8k^3}}$$

$$=\frac{\frac{27}{8k^3}}{\frac{27}{8k^3}}-\frac{\frac{1}{2}}{\frac{27}{8k^3}}=1-\frac{8k^3}{54}=\underline{\underline{1-\frac{4k^3}{27}}}$$

A6 Ausführliche Lösungen

a)

$$a^n+a^{4-n}+a^{2n}$$

$$=a^{2n}\left(\frac{a^n}{a^{2n}}+\frac{a^{4-n}}{a^{2n}}+\frac{a^{2n}}{a^{2n}}\right)$$

$$=\underline{\underline{a^{2n}\left(a^{-n}+a^{4-3n}+1\right)}}$$

b)

$$a^{k-2}+a^{3k}+a^{-k-1}$$

$$=a^{-k}\left(\frac{a^{k-2}}{a^{-k}}+\frac{a^{3k}}{a^{-k}}+\frac{a^{-k-1}}{a^{-k}}\right)$$

$$=\underline{\underline{a^{-k}\left(a^{2k-2}+a^{4k}+a^{-1}\right)}}$$

A6	Ausführliche Lösungen			
	c)	$a^3 + a^{1-n} + a^{n+4}$ $= a^{n+3}\left(\frac{a^3}{a^{n+3}} + \frac{a^{1-n}}{a^{n+3}} + \frac{a^{n+4}}{a^{n+3}}\right)$ $= a^{n+3}\left(a^{-n} + a^{-2n-2} + a\right)$	d)	$\frac{3}{2}x^4 + \frac{3}{4}x^3 + \frac{1}{8}x^2$ $= \frac{1}{8}x^2\left(\frac{\frac{3}{2}x^4}{\frac{1}{8}x^2} + \frac{\frac{3}{4}x^3}{\frac{1}{8}x^2} + \frac{\frac{1}{8}x^2}{\frac{1}{8}x^2}\right)$ $= \frac{1}{8}x^2\left(12x^2 + 6x + 1\right)$

A6	Ausführliche Lösungen			
	e)	$e^{3x} - 2e^{-x} = e^{-x}\left(\frac{e^{3x}}{e^{-x}} - \frac{2e^{-x}}{e^{-x}}\right)$ $= e^{-x}\left(e^{4x} - 2\right)$	f)	$ke^{2x} - 2e^{x+1} = e^{x}\left(\frac{ke^{2x}}{e^{x}} - \frac{2e^{x+1}}{e^{x}}\right)$ $= e^{x}\left(ke^{x} - 2e\right)$