

Lösungen Potenzen V

Ergebnisse:

E1		Ergebnisse	
a)	$\frac{(4-12x)^3}{64} = (1-3x)^3$	b)	$\left(\frac{2}{x^2}\right)^5 - \left(\frac{3}{x^5}\right)^2 = \frac{23}{x^{10}}$
c)	$\left(\left(-\frac{3}{k}\right)^3\right)^4 \cdot \frac{k^9}{81} = \frac{6561}{k^3}$	d)	$\frac{(ab)^2}{x^3y} \cdot \frac{x^5y^2}{a^2b} = bx^2y$

E2		Ergebnisse	
a)	$\frac{1}{4} \cdot 2^4 \cdot (2^2)^3 = 256 = 2^8$		
b)	$(3^{n+1})^2 = 3^{2n+2}$		
c)	$(4x+3y^3)^2 = 16x^2 + 24xy^3 + 9y^6$		
d)	$-(x^4-2)^2 = -x^8 + 4x^4 - 4$		
e)	$(x^2-x^3)(x^2+x^3) = x^4 - x^6$		
f)	$(3x^2+2k)^2 = 9x^4 + 12kx^2 + 4k^2$		
g)	$-\frac{1}{2}(x^2-4)^2 = -\frac{1}{2}x^4 + 4x^2 - 8$		
h)	$\left(-\frac{1}{2}(x^2-4)\right)^2 = \frac{1}{4}x^4 - 2x^2 + 4$		
i)	$x^2y^2(x^4+2x^2y+y^2) = x^6y^2 + 2x^4y^3 + x^2y^4$		

E3		Ergebnisse	
a)	$(3x^2-5x)(1-x^3) + (x^2+3x^4)x^3 = 3x^7 - 2x^5 + 5x^4 + 3x^2 - 5x$		
b)	$a^{2r}b^r(a^{2r}-a^rb^{r+1}+b^{2r+2}) = a^{4r}b^r - a^{3r}b^{2r+1} + a^{2r}b^{3r+2}$		

E4	Ergebnisse
a)	$-3x^3 \cdot x^2 + 5x \cdot x^4 = 2x^5$
b)	$4k^{n-4} \cdot k^3 - k \cdot k^{n-2} = 3k^{n-1}$
c)	$2x^5y^3 \cdot y - 4x^3y^2 \cdot x^2y^2 = -2x^5y^4$
d)	$\frac{4x^5 + 6x^4 - 12x^2}{2x^2} = 2x^3 + 3x^2 - 6$
e)	$(a^{n+2} - 4a^n - 2a^{2-n}) \frac{a^2}{2} = \frac{1}{2}(a^{n+4} - 4a^{n+2} - 2a^{4-n})$
f)	$(9 \cdot 3^n - 3^{n+1}) : 3^{n-1} = 18$

E5	Ergebnisse		
a)	$(2x+6)^2 + (x+3)^2 = 5(x+3)^2$	b)	$\frac{5a-20}{4a-16} = \frac{5}{4}$
c)	$(3k^2 - 3k^3)^2 = 9k^4(k-1)^2$	d)	$\frac{x(5a+15)}{a+3} = 5x$
e)	$\frac{(2x-6)^2}{4} = (x-3)^2$	f)	$\frac{(-2a-4)^3}{a+2} = -8(a+2)^2$

E6	Ergebnisse
a)	$3a^2 + 6a^3 = 3a^2(1+2a)$
b)	$2a^2 - 6a^3 + 4a^4 - 8a^5 = 2a^2(1 - 3a + 2a^2 - 4a^3)$
c)	$2^x + 2^{x+1} = 3 \cdot 2^x$
d)	$\frac{1}{2}e^x - \frac{1}{4}e^{x+1} = \frac{e^x}{4}(2-e)$
e)	$(3x-6) \left(\frac{1}{4}x^2 - x + 1 \right) = \frac{3(x-2)^3}{4}$
f)	$\frac{1}{3}x^3 - 2x^2 + 3x = \frac{x(x-3)^2}{3}$
g)	$a^{5b} + 3a^b = a^b(3 + a^{4b})$
h)	$3x^4 - 12x^2 = 3x^2(x-2)(x+2)$
i)	$a^2 - 2a^3 + a^4 = a^2(a-1)^2$

E7		Ergebnisse	
a)	$x^4 + 2x^3 = x^3(x+2)$	b)	$x^2e^x + 2xe^x + e^x = e^x(x+1)^2$
c)	$x^{n+3} - 4x^{n+2} = x^{n+2}(x-4)$	d)	$3a^3 - 12a^9 = 3a^3(1-2a^3)(1+2a^3)$
e)	$x^{n+2} - 6x^{n+1} + 9x^n = x^n(x-3)^2$	f)	$3-x^2 = (\sqrt{3}-x)(\sqrt{3}+x)$
g)	$x^4 - a^2 = (x^2-a)(x^2+a)$	h)	$-6k^{n+2} + 18k^{2-n} = -6k^2(k^n - 3k^{-n})$
i)	$x^4 - 8x^2 + 12 = (x^2-6)(x^2-2)$	j)	$e^x - e^{3x} = e^x(1-e^{2x})$
k)	$x^{2n} + 4x^n + 4 = (x^n+2)^2$	l)	$e^{2x} - 1 = (e^x-1)(e^x+1)$

Potenzgesetze

$a^m \cdot a^n = a^{m+n}$	$\frac{a^m}{a^n} = a^{m-n}$	$a^n \cdot b^n = (a \cdot b)^n$	$\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n$
$(a^n)^m = a^{n \cdot m}$	$\sqrt[n]{a^m} = a^{\frac{m}{n}}$	$a^0 = 1$	$\frac{1}{a^n} = a^{-n}$

Ausführliche Lösungen :

A1	Ausführliche Lösungen	
	a)	b)
	$\frac{(4-12x)^3}{64} = \frac{[4(1-3x)]^3}{4^3}$ $= \frac{4^3(1-3x)^3}{4^3}$ $= \underline{\underline{(1-3x)^3}}$	$\left(\frac{2}{x^2}\right)^5 - \left(\frac{3}{x^5}\right)^2 = \frac{2^5}{x^{10}} - \frac{3^2}{x^{10}}$ $= \frac{2^5 - 3^2}{x^{10}}$ $= \frac{32-9}{x^{10}} = \underline{\underline{\frac{23}{x^{10}}}}$
	c)	d)
	$\left(\left(-\frac{3}{k}\right)^3\right)^4 \cdot \frac{k^9}{81} = \left[(-1)^3 \frac{3^3}{k^3}\right]^4 \cdot \frac{k^9}{81}$ $= (-1)^{12} \cdot \frac{3^{12}}{k^{12}} \cdot \frac{k^9}{3^4}$ $= 1 \cdot \frac{3^8}{k^3} = \underline{\underline{\frac{6561}{k^3}}}$	$\frac{(ab)^2}{x^3y} \cdot \frac{x^5y^2}{a^2b} = \frac{a^2 \cdot b^2 \cdot x^5 \cdot y^2}{a^2 \cdot b \cdot x^3 \cdot y}$ $= \frac{\cancel{a^2} \cdot b^2 \cdot x^5 \cdot y^2}{\cancel{a^2} \cdot b \cdot x^3 \cdot y}$ $= \underline{\underline{bx^2y}}$

A2	Ausführliche Lösungen	
	a)	
	b)	
	$\frac{1}{4} \cdot 2^4 \cdot (2^2)^3 = \frac{1}{2^2} \cdot 2^4 \cdot 2^6 = 2^8 = \underline{\underline{256}}$	
	$(3^{n+1})^2 = 3^{2 \cdot (n+1)} = \underline{\underline{3^{2n+2}}}$	
	c)	
	$\underbrace{(4x^2 + 3y^3)^2}_{1. \text{ bin. Formel}} = \underline{\underline{16x^2 + 24xy^3 + 9y^6}}$	

A2	Ausführliche Lösungen	
	d)	
	e)	
	$-(x^4 - 2)^2 = (-1) \cdot (x^8 - 4x^4 + 4) = \underline{\underline{-x^8 + 4x^4 - 4}}$ <p>2. bin. Formel</p>	
	$\underbrace{(x^2 - x^3)(x^2 + x^3)}_{3. \text{ bin. Formel}} = \underline{\underline{x^4 - x^6}}$	
	f)	
	$\underbrace{(3x^2 + 2k)^2}_{1. \text{ bin. Formel}} = \underline{\underline{9x^4 + 12kx^2 + 4k^2}}$	

A2	Ausführliche Lösungen	
	g)	
	h)	
	$-\frac{1}{2} \underbrace{(x^2 - 4)^2}_{2. \text{ bin. Formel}} = -\frac{1}{2} (x^4 - 8x^2 + 16) = \underline{\underline{-\frac{1}{2}x^4 + 4x^2 - 8}}$	
	$\left(-\frac{1}{2}(x^2 - 4)\right)^2 = \left(-\frac{1}{2}\right)^2 \underbrace{(x^2 - 4)^2}_{2. \text{ bin. Formel}} = \frac{1}{4} (x^4 - 8x^2 + 16) = \underline{\underline{\frac{1}{4}x^4 - 2x^2 + 4}}$	
	i)	
	$x^2y^2(x^4 + 2x^2y + y^2) = \underline{\underline{x^6y^2 + 2x^4y^3 + x^2y^4}}$	

A3 Ausführliche Lösungen	
a)	$(3x^2 - 5x)(1 - x^3) + (x^2 + 3x^4)x^3$ $= 3x^2 - 3x^5 - 5x + 5x^4 + x^5 + 3x^7$ $= 3x^7 - 3x^5 + x^5 + 5x^4 + 3x^2 - 5x$ $= \underline{\underline{3x^7 - 2x^5 + 5x^4 + 3x^2 - 5x}}$
b)	$a^{2r}b^r (a^{2r} - a^r b^{r+1} + b^{2r+2})$ $= a^{2r} \cdot a^{2r} b^r - a^{2r} \cdot a^r b^r \cdot b^{r+1} + a^{2r} b^r \cdot b^{2r+2}$ $= \underline{\underline{a^{4r} b^r - a^{3r} b^{2r+1} + a^{2r} b^{3r+2}}}$

A4 Ausführliche Lösungen	
a)	$-3x^3 \cdot x^2 + 5x \cdot x^4$ $= -3x^5 + 5x^5$ $= \underline{\underline{2x^5}}$
b)	$4k^{n-4} \cdot k^3 - k \cdot k^{n-2}$ $= 4k^{n-1} - k^{n-1}$ $= \underline{\underline{3k^{n-1}}}$
c)	$2x^5 y^3 \cdot y - 4x^3 y^2 \cdot x^2 y^2$ $= 2x^5 y^4 - 4x^5 y^4$ $= \underline{\underline{-2x^5 y^4}}$
d)	$\frac{4x^5 + 6x^4 - 12x^2}{2x^2}$ $= \frac{4x^5}{2x^2} + \frac{6x^4}{2x^2} - \frac{12x^2}{2x^2}$ $= \underline{\underline{2x^3 + 3x^2 - 6}}$
e)	$(a^{n+2} - 4a^n - 2a^{2-n}) \frac{a^2}{2}$ $= \frac{1}{2} a^2 (a^{n+2} - 4a^n - 2a^{2-n})$ $= \underline{\underline{\frac{1}{2} (a^{n+4} - 4a^{n+2} - 2a^{4-n})}}$
f)	$(9 \cdot 3^n - 3^{n+1}) : 3^{n-1}$ $= \frac{9 \cdot 3^n}{3^{n-1}} - \frac{3^{n+1}}{3^{n-1}}$ $= 3^2 \cdot 3^n \cdot 3^{-n+1} - 3^{n+1-n+1}$ $= \underline{\underline{3^3 - 3^2 = 18}}$

A5 Ausführliche Lösungen	
a)	$(2x+6)^2 + (x+3)^2$ $= [2(x+3)]^2 + (x+3)^2$ $= 2^2(x+3)^2 + (x+3)^2$ $= \underline{\underline{5(x+3)^2}}$
b)	$\frac{5a-20}{4a-16}$ $= \frac{5(a-4)}{4(a-4)}$ $= \frac{5\cancel{(a-4)}}{4\cancel{(a-4)}} = \underline{\underline{\frac{5}{4}}}$

A5 Ausführliche Lösungen	
c)	$(3k^2 - 3k^3)^2 = [3k^2(1-k)]^2$ $= \underline{\underline{9k^4(k-1)^2}}$
d)	$\frac{x(5a+15)}{a+3} = \frac{5x(a+3)}{(a+3)}$ $= \underline{\underline{5x}}$

A5 Ausführliche Lösungen	
e)	$\frac{(2x-6)^2}{4} = \frac{[2(x-3)]^2}{4}$ $= \frac{2^2(x-3)^2}{4}$ $= \underline{\underline{(x-3)^2}}$
f)	$\frac{(-2a-4)^3}{a+2} = \frac{[(-2)(a+2)]^3}{(a+2)}$ $= \frac{(-2)^3(a+2)^3}{(a+2)}$ $= \underline{\underline{-8(a+2)^2}}$

A6 Ausführliche Lösungen	
a)	$3a^2 + 6a^3 = 3a^1 \cdot 1 + 3a^2 \cdot 2a = \underline{\underline{3a^2(1+2a)}}$
b)	$2a^2 - 6a^3 + 4a^4 - 8a^5 = 2a^2 \cdot 1 - 2a^2 \cdot 3a + 2a^2 \cdot 2a^2 - 2a^2 \cdot 4a^3$ $= \underline{\underline{2a^2(1-3a+2a^2-4a^3)}}$
c)	$2^x + 2^{x+1} = 2^x \cdot 1 + 2^x \cdot 2^1 = 2^x(1+2) = \underline{\underline{3 \cdot 2^x}}$

A6 Ausführliche Lösungen	
d)	$\frac{1}{2}e^x - \frac{1}{4}e^{x+1} = \frac{1}{4}e^x \cdot 2 - \frac{1}{4}e^x \cdot e = \underline{\underline{\frac{1}{4}e^x(2-e)}}$
e)	$(3x-6) \left(\frac{1}{4}x^2 - x + 1 \right) = 3(x-2) \cdot \frac{1}{4} \underbrace{(x^2 - 4x + 4)}_{\text{2. bin. Formel}}$ $= 3(x-2) \cdot \frac{1}{4}(x-2)^2$ $= \underline{\underline{\frac{3}{4}(x-2)^3}}$
f)	$\frac{1}{3}x^3 - 2x^2 + 3x = \frac{1}{3}x \cdot x^2 - \frac{1}{3}x \cdot 6x + \frac{1}{3}x \cdot 9$ $= \frac{1}{3}x \underbrace{(x^2 - 6x + 9)}_{\text{2. bin. Formel}}$ $= \underline{\underline{\frac{1}{3}x(x-3)^2}}$

A6 Ausführliche Lösungen	
g)	$a^{5b} + 3a^b = a^b \cdot a^{4b} + a^b \cdot 3 = \underline{\underline{a^b(a^{4b} + 3)}}$
h)	$3x^4 - 12x^2 = 3x^2 \underbrace{(x^2 - 4)}_{\text{3. bin. Formel}} = \underline{\underline{3x^2(x-2)(x+2)}}$
i)	$a^2 - 2a^3 + a^4 = a^2(1 - 2a + a^2) = a^2 \underbrace{(a^2 - 2a + 1)}_{\text{2. bin. Formel}} = \underline{\underline{a^2(a-1)^2}}$

A7 Ausführliche Lösungen	
a) $x^4 + 2x^3 = x^3 \cdot x + x^3 \cdot 2$ $= \underline{\underline{x^3(x+2)}}$	b) $x^2e^x + 2xe^x + e^x = e^x(x^2 + 2x + 1)$ 1. bin. Formel $= \underline{\underline{e^x(x+1)^2}}$
c) $x^{n+3} - 4x^{n+2} = x^{n+2} \cdot x - x^{n+2} \cdot 4$ $= \underline{\underline{x^{n+2}(x-4)}}$	d) $3a^3 - 12a^9 = 3a^3 \cdot 1 - 3a^3 \cdot 4a^6$ $= 3a^3(1 - 4a^6)$ 3. bin. Formel $= \underline{\underline{3a^3(1-2a^3)(1+2a^3)}}$
e) $x^{n+2} - 6x^{n+1} + 9x^n = x^n(x^2 - 6x + 9)$ 2. bin. Formel $= \underline{\underline{x^n(x-3)^2}}$	f) $3 - x^2 = (\sqrt{3} - x)(\sqrt{3} + x)$ 3. bin. Formel
g) $x^4 - a^2 = (x^2 - a)(x^2 + a)$ 3. bin. Formel	h) $-6k^{n+2} + 18k^{2-n} = -6k^2 \cdot k^n - 6k^2 \cdot (-3k^{-n})$ $= \underline{\underline{-6k^2(k^n - 3k^{-n})}}$
i) $x^4 - 8x^2 + 12 = x^4 - 2x^2 - 6x^2 + 2 \cdot 6$ $= \underline{\underline{(x^2 - 6)(x^2 - 2)}}$	j) $e^x - e^{3x} = e^x \cdot 1 - e^x \cdot e^{2x}$ $= \underline{\underline{e^x(1 - e^{2x})}}$
k) $x^{2n} + 4x^n + 4 = (x^n + 2)^2$ 1. bin. Formel	l) $e^{2x} - 1 = (e^x - 1)(e^x + 1)$ 3. bin. Formel