Lösungen Potenzen, Wurzeln und Logarithmen III

Ergebnisse:

E1	Ergebnisse	
	a)	$\sqrt{4}-\sqrt{8}+3\sqrt{18}=2+7\sqrt{2}$
	b)	$x\cdot\ln(y)-y\cdot\ln(x)=\ln\left(\frac{y^x}{x^y}\right)$
	c)	$\ln\left(x^5\right)-\ln\left(x^2\right)+\ln\left(4x^3\right)=\ln\left(4x^6\right)$

E2	Ergebnisse			
	a)	$2^x\cdot\left(\frac{5}{2}\right)^x\cdot 5=5^{x+1}$	b)	$(x-3)^n\cdot(x+3)^n=\left(x^2-9\right)^n$
	c)	$2^n\cdot\left(\frac{x}{2}\right)^n\cdot x=x^{n+1}$	d)	$8x^4-7x^5+2x^4-3x^5=-10\left(x^5-x^4\right)$
	e)	$7x^5-3y^5+x^5-2y^5=8x^5-5y^5$	f)	$5a^m-2a^n+4a^m-3a^n=9a^m-5a^n$

E3	Ergebnisse	
	a)	$\ln\left(2e^2\right)+\ln\left(\frac{e}{2}\right)=3$
	b)	$\ln\left(\frac{4}{3}k\right)-\ln\left(\frac{4}{k}\right)=2\ln(k)-\ln(3);k>0$
	c)	$\ln\left(1-x^2\right)-\ln(1+x)=\ln(1-x)$
	d)	$\ln(x)-\ln(4)+\ln\left(\frac{4y}{x}\right)=\ln(y)$
	e)	$\ln\left(\frac{1}{a^2}\right)-\ln(2a)-\ln\left(\frac{1}{a}\right)=-2\ln(a)-\ln(2)$
	f)	$\ln\frac{1+x}{2+x}-\ln(x+1)=-\ln(2+x)$

E4	Ergebnisse
	a) k^2 b) $\frac{k}{2}$ c) $\frac{1}{2}\sqrt{k}$ d) $\frac{1}{\sqrt[3]{k}}$ e) $2k^2$ f) $k\cdot e^{-1}$ g) $k-1$

E5	Ergebnisse			
	a)	$\frac{\left(4a^2+4a+1\right)^x}{\left(2a+1\right)^x}=\left(2a+1\right)^x$	b)	$\frac{ma^5-ma^3b^2}{ma^2}=a^3-ab^2$
	c)	$\frac{25x^2-100y^2}{5x+10y}=5x-10y$	d)	$\frac{\left(a^2+2ab+b^2\right)}{\left(a+b\right)^3}=\left(a+b\right)^3$
	e)	$\frac{49z^2-1}{7z-1}=7z+1$	f)	$\frac{\left[\left(-x^2\right)^2\right]^2\cdot x^{2a}}{x^6}=x^{2a+2}$

E6	Ergebnisse	
	a)	$\sqrt[3]{u^2}\cdot\sqrt[3]{u^2}\cdot\sqrt[3]{u^5}=u^3$
	b)	$\left(\frac{7xy^2}{3p^2q^2}\right)^5\cdot\left(\frac{18p^2q^2}{14xy^2}\right)^5=3^5=243$
	c)	$\frac{13a^{11}b^3\cdot 14x^4y^9}{26a^{12}b^5}:\frac{x^3y^9}{49ab^2}=343x$
	d)	$3^{-\frac{1}{3}}\cdot\sqrt[3]{\left(-3\right)^4}\cdot\frac{1}{9}=\frac{1}{3}$
	e)	$2\cdot\ln\left(\frac{p}{q}\right)+\frac{1}{2}\cdot\ln\left(q^2\right)-\ln\left(p^3\right)=\ln\left(\frac{1}{pq}\right)$
	f)	$\frac{1}{2}\cdot\ln\left(a\right)-2\cdot\ln\left(b^2\right)+\ln\left(c\right)=\ln\left(\frac{c\cdot\sqrt{a}}{b^4}\right)$

E7	Ergebnisse	
	a)	$\frac{ax+a+x+1}{a^2-1}=\frac{x+1}{a-1}$
	b)	$5\cdot\ln\left(x\right)+\frac{1}{4}\cdot\ln\left(y\right)+\frac{3}{2}\cdot\ln\left(z\right)=\ln\left(x^5\cdot\sqrt[4]{y}\cdot z\cdot\sqrt{z}\right)$
	c)	$4\cdot\ln\left(a\right)-\frac{3}{2}\cdot\ln\left(b^2\right)-\frac{2}{3}\cdot\ln\left(\sqrt{a^3}\right)=\ln\left(\frac{a}{b}\right)^3$
	d)	$\ln\left(\frac{1}{x}\right)+\ln\left(\sqrt{x}\right)-\ln\left(x^3\right)=-\frac{7}{2}\ln\left(x\right)$
	e)	$\ln\left(\frac{5}{x}\right)+\ln\left(\frac{x}{5}\right)-\ln\left(\sqrt[3]{x^2}\right)=-\frac{2}{3}\ln\left(x\right)$
	f)	$\ln\left(\frac{e}{a}\right)+\ln\left(a^2\right)-\ln\left(a\sqrt{e}\right)=\frac{1}{2}$

E8	Ergebnisse	
	a)	$\ln\left(e^3\right) - \ln\left(b^2\right) + 2\cdot\ln\left(\frac{b}{e}\right) = 1$
	b)	$\ln\left(\sqrt{e}\cdot b\right) + \ln\left(\frac{b}{e^2}\right) - \frac{3}{2}\cdot\ln\left(eb\right) = -3$
	c)	$\frac{\ln\left(u^2\right) - \ln\left(u\right) + \ln\left(\sqrt{u}\right)}{\ln\left(u^3\right)} = \frac{1}{2}$

Potenz- Wurzel- und Logarithmengesetze

Potenz- und Wurzelgesetze

$a^m \cdot a^n = a^{m+n}$	$\frac{a^m}{a^n} = a^{m-n}$	$a^n \cdot b^n = (a \cdot b)^n$	$\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n$
$\left(a^n\right)^m = a^{n\cdot m}$	$\sqrt[n]{a^m} = a^{\frac{m}{n}}$	$a^0 = 1$	$\frac{1}{a^n} = a^{-n}$
$\sqrt{a}\cdot\sqrt{b} = \sqrt{a\cdot b}$	$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$	$\left(\sqrt{a}\right)^2 = a$	$\sqrt{a^{-1}} = \sqrt{\frac{1}{a}} = \frac{1}{\sqrt{a}}$

Logarithmus zur Basis a

$x = \log_a(b) \Leftrightarrow a^x = b$	$\log_a(b\cdot c) = \log_a(b) + \log_a(c)$
$\log_a\left(\frac{b}{c}\right) = \log_a(b) - \log_a(c)$	$\log_a\left(b^c\right) = c\cdot\log_a(b)$
$\log_a(a) = 1$ $\quad \log_a(1) = 0$	$b = a^{\log_a(b)}$ $\quad b^x = a^{x\cdot\log_a(b)}$

Logarithmus zur Basis 10 (Zehner- oder dekadischer Logarithmus) [LOG]-Taste

$x = \lg(b) \Leftrightarrow 10^x = b$	$\lg(b\cdot c) = \lg(b) + \lg(c)$
$\lg\left(\frac{b}{c}\right) = \lg(b) - \lg(c)$	$\lg\left(b^c\right) = c\cdot\lg(b)$
$\lg(10) = 1$ $\quad \lg(1) = 0$	$b = 10^{\lg(b)}$ $\quad b^x = 10^{x\cdot\lg(b)}$

Logarithmus zur Basis e (Natürlicher Logarithmus oder Logarithmus Naturalis)

$x = \ln(b) \Leftrightarrow e^x = b$	$\ln(b\cdot c) = \ln(b) + \ln(c)$
$\ln\left(\frac{b}{c}\right) = \ln(b) - \ln(c)$	$\ln\left(b^c\right) = c\cdot\ln(b)$
$\ln(e) = 1$ $\quad \ln(1) = 0$	$b = e^{\ln(b)}$ $\quad b^x = e^{x\cdot\ln(b)}$

Umrechnung von einem Logarithmensystem in ein anderes.

$\log_a(b) = \frac{\lg(b)}{\lg(a)} = \frac{\ln(b)}{\ln(a)}$	$\log_3(5) = \frac{\lg(5)}{\lg(3)} = \frac{\ln(5)}{\ln(3)} \approx 1{,}4649735$

Ausführliche Lösungen:

A1	Ausführliche Lösungen	
	a)	$\sqrt{4}-\sqrt{8}+3\sqrt{18}=2-\sqrt{2\cdot 4}+3\sqrt{2\cdot 9}=2-2\sqrt{2}+9\sqrt{2}=\underline{\underline{2+7\sqrt{2}}}$
	b)	$x\cdot\ln(y)-y\cdot\ln(x)=\ln\left(y^x\right)-\ln\left(x^y\right)=\underline{\underline{\ln\left(\frac{y^x}{x^y}\right)}}$
	c)	$\ln\left(x^5\right)-\ln\left(x^2\right)+\ln\left(4x^3\right)=\ln\left(\frac{x^5\cdot 4x^3}{x^2}\right)=\ln\left(\frac{4x^8}{x^2}\right)=\underline{\underline{\ln\left(4x^6\right)}}$

A2	Ausführliche Lösungen	
	a)	$2^x\cdot\left(\frac{5}{2}\right)^x\cdot 5=2^x\cdot\frac{5^x}{2^x}\cdot 5=5^x\cdot 5=\underline{\underline{5^{x+1}}}$
	b)	$(x-3)^n\cdot(x+3)^n=\left[(x-3)(x+3)\right]^n=\underline{\underline{\left(x^2-9\right)^n}}$
	c)	$2^n\cdot\left(\frac{x}{2}\right)^n\cdot x=2^n\cdot\frac{x^n}{2^n}\cdot x=x^n\cdot x=\underline{\underline{x^{n+1}}}$
	d)	$8x^4-7x^5+2x^4-3x^5=-7x^5-3x^5+8x^4+2x^4=-10x^5+10x^4=\underline{\underline{-10\left(x^5-x^4\right)}}$
	e)	$7x^5-3y^5+x^5-2y^5=7x^5+x^5-3y^5-2y^5=\underline{\underline{8x^5-5y^5}}$
	f)	$5a^m-2a^n+4a^m-3a^n=5a^m+4a^m-2a^n-3a^n=\underline{\underline{9a^m-5a^n}}$

A3	Ausführliche Lösungen
a)	$\ln\left(2e^2\right)+\ln\left(\frac{e}{2}\right)=\ln(2)+2\cdot\ln(e)+\ln(e)-\ln(2)$ $=\ln(2)-\ln(2)+2\cdot\ln(e)+\ln(e)=2+1=\underline{\underline{3}}$
b)	$\ln\left(\frac{4}{3}k\right)-\ln\left(\frac{4}{k}\right);k>0$ $=\ln\left(\frac{\frac{4}{3}k}{\frac{4}{k}}\right)=\ln\left(\frac{4k\cdot k}{3\cdot 4}\right)=\ln\left(\frac{k^2}{3}\right)=\underline{\underline{2\ln(k)-\ln(3)}}$
c)	$\ln\left(1-x^2\right)-\ln(1+x)=\ln\left(\frac{1-x^2}{1+x}\right)=\ln\left[\frac{(1-x)(1+x)}{1+x}\right]=\underline{\underline{\ln(1-x)}}$
d)	$\ln(x)-\ln(4)+\ln\left(\frac{4y}{x}\right)=\ln\left(\frac{x}{4}\cdot\frac{4y}{x}\right)=\underline{\underline{\ln(y)}}$
e)	$\ln\left(\frac{1}{a^2}\right)-\ln(2a)-\ln\left(\frac{1}{a}\right)=\ln\left(a^{-2}\right)-(\ln 2+\ln a)-\left(\underbrace{\ln(1)}_{0}-\ln(a)\right)$ $=-2\cdot\ln(a)-\ln(2)-\ln(a)+\ln(a)=\underline{\underline{-2\cdot\ln(a)-\ln(2)}}$
f)	$\ln\left(\frac{1+x}{2+x}\right)-\ln(x+1)=\ln\left[\frac{1+x}{(2+x)(x+1)}\right]=\ln\left(\frac{1}{2+x}\right)$ $=\ln(1)-\ln(2+x)=\underline{\underline{-\ln(2+x)}}$

A4	Ausführliche Lösungen
a)	$e^{2\ln(k)}=\left(e^{\ln(k)}\right)^2=\underline{\underline{k^2}}$
b)	$e^{\ln\left(\frac{k}{2}\right)}=\underline{\underline{\frac{k}{2}}}$
c)	$\frac{1}{2}e^{\frac{\ln(k)}{2}}=\frac{1}{2}e^{\frac{1}{2}\cdot\ln(k)}=\frac{1}{2}\left(e^{\ln(k)}\right)^{\frac{1}{2}}=\frac{1}{2}k^{\frac{1}{2}}=\underline{\underline{\frac{1}{2}\sqrt{k}}}$
d)	$e^{-\frac{\ln(k)}{3}}=\frac{1}{e^{\frac{\ln(k)}{3}}}=\frac{1}{e^{\frac{1}{3}\cdot\ln(k)}}=\frac{1}{\left(e^{\ln(k)}\right)^{\frac{1}{3}}}=\frac{1}{k^{\frac{1}{3}}}=\underline{\underline{\frac{1}{\sqrt[3]{k}}}}$
e)	$2\cdot e^{\ln\left(k^2\right)}=2\cdot k^2$
f)	$e^{\ln k-1}=\frac{e^{\ln(k)}}{e}=\frac{k}{e}=\underline{\underline{k\cdot e^{-1}}}$
g)	$e^{\ln(k-1)}=\underline{\underline{k-1}}$

A5 Ausführliche Lösungen

a)
$$\frac{\left(4a^2+4a+1\right)^x}{(2a+1)^x}=\left(\frac{4a^2+4a+1}{2a+1}\right)^x=\left[\frac{(2a+1)^2}{2a+1}\right]^x=\underline{\underline{(2a+1)^x}}$$

b)
$$\frac{ma^5-ma^3b^2}{ma^2}=\frac{ma^2\left(a^3-ab^2\right)}{ma^2}=\underline{\underline{a^3-ab^2}}$$

c)
$$\frac{25x^2-100y^2}{5x+10y}=\frac{(5x+10y)(5x-10y)}{5x+10y}=\underline{\underline{5x-10y}}$$

d)
$$\frac{\left(a^2+2ab+b^2\right)^3}{(a+b)^3}=\left(\frac{a^2+2ab+b^2}{a+b}\right)^3=\left[\frac{(a+b)^2}{a+b}\right]^3=\underline{\underline{(a+b)^3}}$$

e)
$$\frac{49z^2-1}{7z-1}=\frac{(7z+1)(7z-1)}{7z-1}=\underline{\underline{7z+1}}$$

f)
$$\frac{\left[\left(-x^2\right)^2\right]^2\cdot x^{2a}}{x^6}=\frac{\left[x^4\right]^2\cdot x^{2a}}{x^6}=\frac{x^8\cdot x^{2a}}{x^6}=x^2\cdot x^{2a}=\underline{\underline{x^{2a+2}}}$$

A6 Ausführliche Lösungen

a)
$$\sqrt[3]{u^2}\cdot\sqrt[3]{u^2}\cdot\sqrt[3]{u^5}=u^{\frac{2}{3}}\cdot u^{\frac{2}{3}}\cdot u^{\frac{5}{3}}=u^{\frac{2}{3}+\frac{2}{3}+\frac{5}{3}}=u^{\frac{9}{3}}=\underline{\underline{u^3}}$$

b)
$$\left(\frac{7xy^2}{3p^2q^2}\right)^5\cdot\left(\frac{18p^2q^2}{14xy^2}\right)^5=\left(\frac{7xy^2\cdot 18p^2q^2}{3p^2q^2\cdot 14xy^2}\right)^5=\left(\frac{\cancel{7xy^2}\cdot 3\cdot\cancel{2}\cdot\cancel{3p^2q^2}}{\cancel{3p^2q^2}\cdot\cancel{2}\cdot\cancel{7xy^2}}\right)^5=3^5=\underline{\underline{243}}$$

c)
$$\frac{13a^{11}b^3\cdot 14x^4y^9}{26a^{12}b^5}:\frac{x^3y^9}{49ab^2}=\frac{13a^{11}b^3\cdot 14x^4\cancel{y^9}\cdot 49ab^2}{26a^{12}b^5\cdot x^3\cancel{y^9}}$$
$$=\frac{13\cdot 14\cdot 49\cdot\cancel{a^{12}}\cancel{b^5}x^4}{26\cdot\cancel{a^{12}}\cancel{b^5}x^3}=\frac{\cancel{13}\cdot\cancel{2}\cdot 7\cdot 49\cdot x^4}{\cancel{2}\cdot\cancel{13}\cdot x^3}=\underline{\underline{343x}}$$

d)
$$3^{-\frac{1}{3}}\cdot\sqrt[3]{(-3)^4}\cdot\frac{1}{9}=3^{-3}\cdot 3^{\frac{4}{3}}\cdot 3^{-2}=3^{-\frac{1}{3}+\frac{4}{3}-\frac{6}{3}}=3^{-\frac{3}{3}}=3^{-1}=\underline{\underline{\frac{1}{3}}}$$

e)
$$2\cdot\ln\left(\frac{p}{q}\right)+\frac{1}{2}\cdot\ln\left(q^2\right)-\ln\left(p^3\right)=\ln\left(\frac{p}{q}\right)^2+\ln\left(q^2\right)^{\frac{1}{2}}-\ln\left(p^3\right)$$
$$=\ln\left(\frac{p}{q}\right)^2+\ln(q)-\ln\left(p^3\right)=\ln\left(\frac{p^2\cdot q}{q\cdot p^3}\right)=\underline{\underline{\ln\left(\frac{1}{pq}\right)}}$$

f)
$$\frac{1}{2}\cdot\ln(a)-2\cdot\ln\left(b^2\right)+\ln(c)=\ln\left(a^{\frac{1}{2}}\right)-\ln\left(b^4\right)+\ln(c)$$
$$=\ln\left(\frac{a^{\frac{1}{2}}}{b^4}\cdot c\right)=\underline{\underline{\ln\left(\frac{c\cdot\sqrt{a}}{b^4}\right)}}$$

A7	Ausführliche Lösungen	
	a)	$\frac{ax+a+x+1}{a^2-1}=\frac{a(x+1)+1\cdot(x+1)}{(a+1)(a-1)}=\frac{(x+1)(a+1)}{(a+1)(a-1)}=\underline{\underline{\frac{x+1}{a-1}}}$
	b)	$5\cdot\ln(x)+\frac{1}{4}\cdot\ln(y)+\frac{3}{2}\cdot\ln(z)=\ln\left(x^5\right)+\ln\left(y^{\frac{1}{4}}\right)+\ln\left(z^{\frac{3}{2}}\right)$ $=\ln\left(x^5\cdot\sqrt[4]{y}\cdot\sqrt{z^3}\right)=\underline{\underline{\ln\left(x^5\cdot\sqrt[4]{y}\cdot z\cdot\sqrt{z}\right)}}$
	c)	$4\cdot\ln(a)-\frac{3}{2}\cdot\ln\left(b^2\right)-\frac{2}{3}\cdot\ln\left(\sqrt{a^3}\right)=\ln\left(a^4\right)-\ln\left(b^2\right)^{\frac{3}{2}}-\ln\left(a^{\frac{3}{2}}\right)^{\frac{2}{3}}$ $=\ln\left(a^4\right)-\ln\left(b^3\right)-\ln(a)=\ln\left(\frac{a^4}{b^3a}\right)=\ln\left(\frac{a^3}{b^3}\right)=\underline{\underline{\ln\left(\frac{a}{b}\right)^3}}$
	d)	$\ln\left(\frac{1}{x}\right)+\ln\left(\sqrt{x}\right)-\ln\left(x^3\right)=\ln\left(x^{-1}\right)+\ln\left(x^{\frac{1}{2}}\right)-\ln\left(x^3\right)=\ln\left(\frac{\sqrt{x}}{x\cdot x^3}\right)$ $=\ln\left(\frac{\sqrt{x}}{x^4}\right)=\ln\left(x^{\frac{1}{2}-4}\right)=\ln\left(x^{-\frac{7}{2}}\right)=\underline{\underline{-\frac{7}{2}\ln(x)}}$
	e)	$\ln\left(\frac{5}{x}\right)+\ln\left(\frac{x}{5}\right)-\ln\left(\sqrt[3]{x^2}\right)=\ln\left(\frac{5x}{x\cdot 5x^{\frac{2}{3}}}\right)=\ln\left(\frac{1}{x^{\frac{2}{3}}}\right)=\ln\left(x^{-\frac{2}{3}}\right)=\underline{\underline{-\frac{2}{3}\ln(x)}}$
	f)	$\ln\left(\frac{e}{a}\right)+\ln\left(a^2\right)-\ln\left(a\sqrt{e}\right)=\ln\left(\frac{e\cdot a^2}{a\cdot a\cdot e^{\frac{1}{2}}}\right)=\ln\left(\frac{e}{e^{\frac{1}{2}}}\right)$ $=\ln\left(e^{1-\frac{1}{2}}\right)=\ln\left(e^{\frac{1}{2}}\right)=\frac{1}{2}\cdot\ln(e)=\underline{\underline{\frac{1}{2}}}$

A8	Ausführliche Lösungen
a)	$\ln\left(e^3\right)-\ln\left(b^2\right)+2\cdot\ln\left(\frac{b}{e}\right)=\ln\left(e^3\right)-\ln\left(b^2\right)+\ln\left(\frac{b}{e}\right)^2$ $=\ln\left[\frac{e^3\cdot\left(\frac{b}{e}\right)^2}{b^2}\right]=\ln\left(\frac{e^3\cdot b^2}{e^2\cdot b^2}\right)=\ln e=\underline{\underline{1}}$
b)	$\ln\left(\sqrt{e\cdot b}\right)+\ln\left(\frac{b}{e^2}\right)-\frac{3}{2}\cdot\ln\left(eb\right)=\ln\left(eb\right)^{\frac{1}{2}}+\ln\left(b\cdot e^{-2}\right)-\ln\left(eb\right)^{\frac{3}{2}}$ $=\ln\left(\frac{e^{\frac{1}{2}}\cdot b^{\frac{1}{2}}\cdot b\cdot e^{-2}}{e^{\frac{3}{2}}\cdot b^{\frac{3}{2}}}\right)=\ln\left(e^{\frac{1}{2}}\cdot e^{-2}\cdot e^{-\frac{3}{2}}\cdot b^{\frac{1}{2}}\cdot b\cdot b^{-\frac{3}{2}}\right)=\ln\left(e^{\frac{1}{2}-2-\frac{3}{2}}\cdot b^{\frac{1}{2}+1-\frac{3}{2}}\right)$ $=\ln\left(e^{-3}\cdot b^0\right)=\ln\left(e^{-3}\cdot 1\right)=-3\cdot\ln\left(e\right)=\underline{\underline{-3}}$
c)	$\frac{\ln\left(u^2\right)-\ln\left(u\right)+\ln\left(\sqrt{u}\right)}{\ln\left(u^3\right)}=\frac{\ln\left(\frac{u^2\cdot u^{\frac{1}{2}}}{u}\right)}{\ln\left(u^3\right)}=\frac{\ln\left(u\cdot u^{\frac{1}{2}}\right)}{\ln\left(u^3\right)}=\frac{\ln\left(u^{\frac{3}{2}}\right)}{\ln\left(u^3\right)}=\frac{\frac{3}{2}\ln\left(u\right)}{3\ln\left(u\right)}=\frac{\frac{3}{2}}{3}=\underline{\underline{\frac{1}{2}}}$