Lösungen Lineare Gleichungen I

Ergebnisse:

E1	Ergebnisse			
	a)	$x-5=9 \Rightarrow L=\{14\}$	b)	$35+x=84 \Rightarrow L=\{49\}$
	c)	$x-6=13 \Rightarrow L=\{19\}$	d)	$8+x=25 \Rightarrow L=\{17\}$

E2	Ergebnisse			
	a)	$x+28=46 \Rightarrow L=\{18\}$	b)	$125+x=264 \Rightarrow L=\{139\}$
	c)	$x-\frac{2}{3}=\frac{3}{4} \Rightarrow L=\left\{\frac{17}{12}\right\}$	d)	$25\frac{1}{3}-x=8\frac{5}{6} \Rightarrow L=\left\{\frac{33}{2}\right\}$

E3	Ergebnisse			
	a)	$\frac{5}{9}=x-\frac{1}{3} \Rightarrow L=\left\{\frac{8}{9}\right\}$	b)	$1,2-x=0,75 \Rightarrow L=\{0,45\}$
	c)	$u=v+x \Rightarrow L=\{u-v\}$	d)	$m=x-b \Rightarrow L=\{m+b\}$

E4	Ergebnisse	
	a)	$a+b+x=a+p \Rightarrow L=\{p-b\}$
	b)	$m+n=x+m-a \Rightarrow L=\{a+n\}$
	c)	$5a^2+2a+x=6a^2+3a \Rightarrow L=\{a^2+a\}$
	d)	$0,4a+x-1,2b=0,8a-0,8b-0,4a \Rightarrow L=\{0,4b\}$

E5	Ergebnisse	
	a)	$3x=3 \Rightarrow L=\{1\}$
	b)	$3x=\frac{3}{5} \Rightarrow L=\left\{\frac{1}{5}\right\}$
	c)	$\frac{4}{5}x=2,4 \Rightarrow L=\{3\}$
	d)	$88=4x-16 \Rightarrow L=\{26\}$

E6	Ergebnisse	
	a)	$mx=m \Rightarrow L=\{1\}$
	b)	$px-p=p \Rightarrow L=\{2\}$
	c)	$a+bx=3b+a \Rightarrow L=\{3\}$

E7	Ergebnisse
a)	$3n-4mx=3n-2m \Rightarrow L=\left\{\frac{1}{2}\right\}$
b)	$6a-5b=8a-3b-ax \Rightarrow L=\left\{\frac{2a+2b}{a}\right\};\ a\neq 0$

E8	Ergebnisse
a)	$5a^2b^3-a^2b+a^2bx=6a^2b^3+a^2b \Rightarrow L=\left\{b^2+2\right\}$
b)	$6abc-5rst-4a^2b^2cx=5abc-3rst-3a^2b^2cx-2rst$ $\Rightarrow L=\left\{\frac{1}{ab}\right\};\quad a\neq 0;\quad b\neq 0$

E9	Ergebnisse
a)	$\frac{x}{4}=5 \Rightarrow L=\{20\}$
b)	$\frac{2x}{3}=4 \Rightarrow L=\{6\}$
c)	$\frac{b^2m^3x}{n}=m^3 \Rightarrow L=\left\{\frac{n}{b^2}\right\};\quad n\neq 0; b\neq 0$
d)	$\frac{a^2bc}{x}=ac \Rightarrow L=\{ab\};\quad a\neq 0; b\neq 0$

E10	Ergebnisse
a)	$\frac{x}{a^2b}=ab^2 \Rightarrow L=\left\{a^3b^3\right\};\quad a\neq 0; b\neq 0$
b)	$\frac{abc^2}{x}=ac \Rightarrow L=\{bc\}$
c)	$\frac{2}{3}+\frac{1}{3}x=\frac{4}{5} \Rightarrow L=\left\{\frac{2}{5}\right\}$
d)	$\frac{3}{8}=\frac{4}{5}-x \Rightarrow L=\left\{\frac{17}{40}\right\}$

E11	Ergebnisse
a)	$\frac{2}{3}x-\frac{5}{6}+\frac{1}{2}x-\frac{3}{8}=\frac{4}{5}x-\frac{3}{4} \Rightarrow L=\left\{1\frac{1}{4}\right\}$
b)	$\frac{2a^2bx}{3bx}=a^2x \Rightarrow L=\left\{\frac{2}{3}\right\};\quad b\neq 0$
c)	$\frac{1}{6}x-4=2 \Rightarrow L=\{36\}$

E12	Ergebnisse	
	a)	$\frac{3}{4}x-2=\frac{2}{5}x+\frac{1}{10}\Rightarrow L=\{6\}$
	b)	$\frac{x}{2}-\frac{3}{4}=\frac{2x}{3}-\frac{5}{12}\Rightarrow L=\{-2\}$
	c)	$\frac{2ax}{3}+\frac{a}{2}=\frac{ax}{4}-\frac{3a}{4}\Rightarrow L=\{-3\}$
	d)	$\frac{abx}{2}+\frac{7a}{12}=\frac{2abx}{3}+\frac{5a}{12}\Rightarrow L=\left\{\frac{1}{b}\right\};\ b\neq 0$

Ausführliche Lösungen:

A1	Ausführliche Lösungen			
	a)	$x-5=9 \mid +5$ $\Leftrightarrow x=14$ $\Rightarrow L=\{14\}$	b)	$35+x=84 \mid -35$ $\Leftrightarrow x=49$ $\Rightarrow L=\{49\}$
	c)	$x-6=13 \mid +6$ $\Leftrightarrow x=19$ $\Rightarrow L=\{19\}$	d)	$8+x=25 \mid -8$ $\Leftrightarrow x=17$ $\Rightarrow L=\{17\}$

A2	Ausführliche Lösungen			
	a)	$x+28=46 \mid -28$ $\Leftrightarrow x=18$ $\Rightarrow L=\{18\}$	b)	$125+x=264 \mid -125$ $\Leftrightarrow x=139$ $\Rightarrow L=\{139\}$
	c)	$x-\frac{2}{3}=\frac{3}{4} \mid +\frac{2}{3}$ $\Leftrightarrow x=\frac{3}{4}+\frac{2}{3} \quad HN=12$ $\Leftrightarrow x=\frac{9}{12}+\frac{8}{12}=\frac{17}{12}$ $\Rightarrow L=\left\{\frac{17}{12}\right\}$	d)	$25\frac{1}{3}-x=8\frac{5}{6}$ $\Leftrightarrow \frac{76}{3}-x=\frac{53}{6} \mid -\frac{76}{3}$ $\Leftrightarrow -x=\frac{53}{6}-\frac{76}{3} \quad HN=6$ $\Leftrightarrow -x=\frac{53}{6}-\frac{152}{6}=-\frac{99}{6} \mid \cdot(-1)$ $\Leftrightarrow x=\frac{99}{6}=\frac{33}{2}$ $\Rightarrow L=\left\{\frac{33}{2}\right\}$

A3	Ausführliche Lösungen			
	a)	$\frac{5}{9}=x-\frac{1}{3} \mid +\frac{1}{3}$ $\Leftrightarrow \frac{5}{9}+\frac{1}{3}=x$ $\Leftrightarrow x=\frac{5}{9}+\frac{1}{3} \quad HN:9$ $\Leftrightarrow x=\frac{5}{9}+\frac{3}{9}=\frac{8}{9} \Rightarrow L=\left\{\frac{8}{9}\right\}$	b)	$1,2-x=0,75 \mid +x$ $\Leftrightarrow 1,2=x+0,75 \mid -0,75$ $\Leftrightarrow 0,45=x$ $\Leftrightarrow x=0,45$ $\Rightarrow L=\{0,45\}$
	c)	$u=v+x \mid -v$ $\Leftrightarrow u-v=x$ $\Leftrightarrow x=u-v$ $\Rightarrow L=\{u-v\}$	d)	$m=x-b \mid +b$ $\Leftrightarrow m+b=x$ $\Leftrightarrow x=m+b$ $\Rightarrow L=\{m+b\}$

A4 Ausführliche Lösungen

a)
$$a+b+x=a+p \mid -a$$
$$\Leftrightarrow b+x=p \mid -b$$
$$\Leftrightarrow x=p-b$$
$$\Rightarrow L=\{p-b\}$$

b)
$$m+n=x+m-a \mid -m$$
$$\Leftrightarrow n=x-a \mid +a$$
$$\Leftrightarrow n+a=x$$
$$\Leftrightarrow x=a+n$$
$$\Rightarrow L=\{a+n\}$$

c)
$$5a^2+2a+x=6a^2+3a \mid -5a^2$$
$$\Leftrightarrow 2a+x=a^2+3a \mid -2a$$
$$\Leftrightarrow x=a^2-a$$
$$\Rightarrow L=\{a^2+a\}$$

d)
$$0,4a+x-1,2b=0,8a-0,8b-0,4a$$
$$\Leftrightarrow 0,4a+x-1,2b=0,4a-0,8b \mid -0,4a$$
$$\Leftrightarrow x-1,2b=-0,8a \mid +1,2b$$
$$\Leftrightarrow x=0,4b$$
$$\Rightarrow L=\{0,4b\}$$

A5 Ausführliche Lösungen

a)
$$3x=3 \mid :3$$
$$\Leftrightarrow x=1$$
$$\Rightarrow L=\{1\}$$

b)
$$3x=\frac{3}{5} \mid :3$$
$$\Leftrightarrow x=\frac{1}{5}$$
$$\Rightarrow L=\left\{\frac{1}{5}\right\}$$

c)
$$\frac{4}{5}x=2,4=\frac{24}{10}=\frac{12}{5} \mid \cdot\frac{5}{4}$$
$$\Leftrightarrow x=\frac{12}{5}\cdot\frac{5}{4}=\frac{12}{4}=3$$
$$\Rightarrow L=\{3\}$$

d)
$$88=4x-16 \mid +16$$
$$\Leftrightarrow 104=4x \mid :4$$
$$\Leftrightarrow 26=x$$
$$\Leftrightarrow x=26$$
$$\Rightarrow L=\{26\}$$

A6 Ausführliche Lösungen

a) $mx=m \mid :m \Leftrightarrow x=1 \Rightarrow L=\{1\}$

b) $px-p=p \mid +p \Leftrightarrow px=2p \mid :p \Leftrightarrow x=2 \Rightarrow L=\{2\}$

c) $a+bx=3b+a \mid -a \Leftrightarrow bx=3b \mid :b \Leftrightarrow x=3 \Rightarrow L=\{3\}$

A7 Ausführliche Lösungen

a)
$$3n-4mx=3n-2m \mid -3n$$
$$\Leftrightarrow -4mx=-2m \mid :m$$
$$\Leftrightarrow -4x=-2 \mid :(-4)$$
$$\Leftrightarrow x=\frac{-2}{-4}=\frac{2}{4}=\frac{1}{2}$$
$$\Rightarrow L=\left\{\frac{1}{2}\right\}$$

b)
$$6a-5b=8a-3b-ax \mid -8a$$
$$\Leftrightarrow -2a-5b=-3b-ax \mid +3b$$
$$\Leftrightarrow -2a-2b=-ax \mid :(-a)$$
$$\Leftrightarrow \frac{-2a-2b}{-a}=x$$
$$\Leftrightarrow x=\frac{2a+2b}{a}$$
$$\Rightarrow L=\left\{\frac{2a+2b}{a}\right\};\ a\neq 0$$

A8 Ausführliche Lösungen

a)

$$5a^2b^3 - a^2b + a^2bx = 6a^2b^3 + a^2b \mid -5a^2b^3$$

$$\Leftrightarrow \quad -a^2b + a^2bx = a^2b^3 + a^2b \mid +a^2b$$

$$\Leftrightarrow \quad a^2bx = a^2b^3 + 2a^2b \mid : a^2b$$

$$\Leftrightarrow x = \frac{a^2b^3 + 2a^2b}{a^2b} = \frac{\cancel{a^2b}\left(b^2+2\right)}{\cancel{a^2b}} = b^2 + 2$$

$$\Rightarrow L = \left\{b^2 + 2\right\}$$

b)

$$\underbrace{6abc - 5rst - 4a^2b^2cx}_{\text{ordnen}} = \underbrace{5abc - 3rst - 3a^2b^2cx - 2rst}_{\text{ordnen und zusammenfassen}}$$

$$\Leftrightarrow -4a^2b^2cx + 6abc - 5rst = -3a^2b^2cx + 5abc - 5rst \mid +3a^2b^2cx$$

$$\Leftrightarrow -a^2b^2cx + 6abc - 5rst = 5abc - 5rst \mid +5rst$$

$$\Leftrightarrow -a^2b^2cx + 6abc = 5abc \mid -6abc$$

$$\Leftrightarrow -a^2b^2cx = -abc \mid : -a^2b^2c$$

$$\Leftrightarrow x = \frac{-abc}{-a^2b^2c} = \frac{\cancel{-abc}\,(1)}{\cancel{-abc}\,(ab)} = \frac{1}{ab}$$

$$\Rightarrow L = \left\{\frac{1}{ab}\right\}; \quad a \neq 0; \quad b \neq 0$$

A9 Ausführliche Lösungen

a)

$$\frac{x}{4} = 5 \mid \cdot 4$$

$$\Leftrightarrow x = 20$$

$$\Rightarrow L = \{20\}$$

b)

$$\frac{2x}{3} = 4 \mid \cdot \frac{3}{2}$$

$$\Leftrightarrow x = \frac{4}{1} \cdot \frac{3}{2} = \frac{12}{2} = 6$$

$$\Rightarrow L = \{6\}$$

c)

$$\frac{b^2m^3x}{n} = m^3 \mid \cdot n$$

$$\Leftrightarrow b^2m^3x = m^3n \mid : b^2m^3$$

$$\Leftrightarrow x = \frac{\cancel{m^3}\,n}{b^2\,\cancel{m^3}} = \frac{n}{b^2}$$

$$\Rightarrow L = \left\{\frac{n}{b^2}\right\}; \quad n \neq 0; b \neq 0$$

d)

$$\frac{a^2bc}{x} = ac \mid \cdot x$$

$$\Leftrightarrow a^2bc = acx$$

$$\Leftrightarrow acx = a^2bc \mid : ac$$

$$\Leftrightarrow x = \frac{a^2bc}{ac} = ab$$

$$\Rightarrow L = \{ab\}; \quad a \neq 0; b \neq 0$$

A10 Ausführliche Lösungen

a)

$$\frac{x}{a^2b} = ab^2 \mid \cdot a^2b$$
$$\Leftrightarrow x = ab^2 \cdot a^2b$$
$$\Leftrightarrow x = a^3b^3$$
$$\Rightarrow L = \{a^3b^3\};\quad a \neq 0; b \neq 0$$

b)

$$\frac{abc^2}{x} = ac \mid \cdot x$$
$$\Leftrightarrow abc^2 = acx$$
$$\Leftrightarrow acx = abc^2 \mid : ac$$
$$\Leftrightarrow x = \frac{abc^2}{ac} = bc$$
$$\Rightarrow L = \{bc\}$$

c)

$$\frac{2}{3} + \frac{1}{3}x = \frac{4}{5} \mid -\frac{2}{3}$$
$$\Leftrightarrow \frac{1}{3}x = \frac{4}{5} - \frac{2}{3} \mid \cdot 3$$
$$\Leftrightarrow x = \frac{12}{5} - \frac{6}{3} = \frac{12}{5} - 2 \mid HN = 5$$
$$\Leftrightarrow x = \frac{12}{5} - \frac{10}{5} = \frac{2}{5}$$
$$\Rightarrow L = \left\{\frac{2}{5}\right\}$$

d)

$$\frac{3}{8} = \frac{4}{5} - x \mid +x$$
$$\Leftrightarrow x + \frac{3}{8} = \frac{4}{5} \mid -\frac{3}{8}$$
$$\Leftrightarrow x = \frac{4}{5} - \frac{3}{8} \mid HN = 40$$
$$\Leftrightarrow x = \frac{32}{40} - \frac{15}{40} = \frac{17}{40}$$
$$\Rightarrow L = \left\{\frac{17}{40}\right\}$$

A11 Ausführliche Lösungen

a)

$$\frac{2}{3}x - \frac{5}{6} + \frac{1}{2}x - \frac{3}{8} = \frac{4}{5}x + \frac{3}{4} \mid -\frac{4}{5}x$$
$$\Leftrightarrow \underbrace{\frac{2}{3}x + \frac{1}{2}x - \frac{4}{5}x}_{HN=30} - \frac{5}{6} - \frac{3}{8} = \frac{3}{4} \mid +\frac{5}{6} + \frac{3}{8}$$
$$\Leftrightarrow \frac{20}{30}x + \frac{15}{30}x - \frac{24}{30}x = \underbrace{\frac{3}{4} + \frac{5}{6} + \frac{3}{8}}_{HN=48}$$
$$\Leftrightarrow \frac{11}{30}x = \frac{36}{48} + \frac{40}{48} + \frac{18}{48} = \frac{22}{48} \mid \cdot \frac{30}{11}$$
$$\Leftrightarrow x = \frac{22 \cdot 30}{48 \cdot 11} = \frac{2 \cdot 30}{4 \cdot 12} = \frac{15}{12} = \frac{5}{4}$$
$$\Rightarrow L = \left\{\frac{5}{4}\right\}$$

b)

$$\frac{2a^2 \not{b}\, \not{x}}{3\not{b}\, \not{x}} = a^2x \Leftrightarrow \frac{2a^2}{3} = a^2x$$
$$\Leftrightarrow a^2x = \frac{2a^2}{3} \mid : a^2 \Leftrightarrow x = \frac{2}{3} \Rightarrow L = \left\{\frac{2}{3}\right\};\quad b \neq 0$$

c)

$$\frac{1}{6}x - 4 = 2 \mid +4 \Leftrightarrow \frac{1}{6}x = 6 \mid \cdot 6 \Leftrightarrow x = 36 \Rightarrow L = \{36\}$$

A12 Ausführliche Lösungen

a)

$$\frac{3}{4}x-2=\frac{2}{5}x+\frac{1}{10}\;|-\frac{2}{5}x$$

$$\Leftrightarrow \underbrace{\frac{3}{4}x-\frac{2}{5}x}_{HN=20}-2=\frac{1}{10}\;|+2$$

$$\Leftrightarrow \frac{15}{20}x-\frac{8}{20}x=\frac{1}{10}+\frac{20}{10}$$

$$\Leftrightarrow \frac{7}{20}x=\frac{21}{10}\;|\cdot\frac{20}{7}$$

$$\Leftrightarrow x=\frac{21}{10}\cdot\frac{20}{7}=6$$

$$\Rightarrow L=\{6\}$$

b)

$$\frac{x}{2}-\frac{3}{4}=\frac{2x}{3}-\frac{5}{12}\;|-\frac{2x}{3}$$

$$\Leftrightarrow \underbrace{\frac{x}{2}-\frac{2x}{3}}_{HN=6}-\frac{3}{4}=-\frac{5}{12}\;|+\frac{3}{4}$$

$$\Leftrightarrow \frac{3x}{6}-\frac{4x}{6}=\underbrace{-\frac{5}{12}+\frac{3}{4}}_{HN=12}$$

$$\Leftrightarrow -\frac{x}{6}=-\frac{5}{12}+\frac{9}{12}=\frac{4}{12}=\frac{1}{3}\;|\cdot(-6)$$

$$\Leftrightarrow x=-\frac{6}{3}=-3\Rightarrow L=\{-2\}$$

c)

$$\frac{2ax}{3}+\frac{a}{2}=\frac{ax}{4}-\frac{3a}{4}\;|-\frac{ax}{4}$$

$$\Leftrightarrow \underbrace{\frac{2ax}{3}-\frac{ax}{4}}_{HN=12}+\frac{a}{2}=-\frac{3a}{4}\;|-\frac{a}{2}$$

$$\Leftrightarrow \frac{8ax}{12}-\frac{3ax}{12}=\underbrace{-\frac{3a}{4}-\frac{a}{2}}_{HN=4}$$

$$\Leftrightarrow \frac{5ax}{12}=-\frac{3a}{4}-\frac{2a}{4}=-\frac{5a}{4}\;|\cdot 12$$

$$\Leftrightarrow 5ax=-\frac{60a}{4}=-15a\;|:5a$$

$$\Leftrightarrow x=-3\Rightarrow L=\{-3\}$$

d)

$$\frac{abx}{2}+\frac{7a}{12}=\frac{2abx}{3}+\frac{5a}{12}\;|-\frac{2abx}{3}$$

$$\Leftrightarrow \underbrace{\frac{abx}{2}-\frac{2abx}{3}}_{HN=6}+\frac{7a}{12}=\frac{5a}{12}\;|-\frac{7a}{12}$$

$$\Leftrightarrow \frac{3abx}{6}-\frac{4abx}{6}=\frac{5a}{12}-\frac{7a}{12}$$

$$\Leftrightarrow -\frac{abx}{6}=-\frac{2a}{12}=-\frac{a}{6}\;|\cdot(-6)$$

$$\Leftrightarrow abx=a\;|:(ab)$$

$$\Leftrightarrow x=\frac{a}{ab}=\frac{1}{b}\Rightarrow L=\left\{\frac{1}{b}\right\};\ b\neq 0$$